Available online at http://iorajournal.org/indx.php/orics /index

Operations Research: International Conference Series

e-ISSN: 2722-0974
p-ISSN: 2723-1739

Vol. 4, No. 3, pp. 87-92, 2023

Determining the Optimal Portfolio Markowitz Model on Moving Stock Prices Using Brown Motion Geometry

Fikrianto Suhariman[1*], Agung Prabowo[2], Ari Wardayani[3], Usman Abbas Yakubu[4]

[1,2,3] *Department of Mathematics, FMIPA, Jenderal Soedirman University*
[4]*Department of Mathematics, Yusuf Maitama Sule University, Kano, Nigeria*

**Corresponding author email:agung.prabowo@unsoed.ac.id*

Abstract

Stock price movements that fluctuate and follow a stochastic process will make it difficult for investors to start investing. For that, we need a stochastic mathematical model. The Brownian geometry motion model is one of the stochastic models that can be used to display the condition of a stock's price movement. Investment is related to the rate of return (return) and the risk obtained. The higher the rate of return obtained, the higher the risk obtained. Therefore, a portfolio calculation is needed, one of which is using the Markowitz model. The Markowitz model can be used to determine the optimal portfolio. The purpose of this study is to model stock prices and form an optimal portfolio. The stock price data used are BBRI, TLKM, and ADRO for the period from 1 July 2021 to 31 August 2022. The results obtained from this study are that there are three portfolio preferences. If investors like high risk to get high returns, then the combined allocation of funds for BBRI, TLKM and ADRO shares is 10.51%, 42.05% and 47.44% respectively. If investors do not like high risk but still want to get a return that is balanced with risk, then the combination of fund allocation for shares of BBRI, TLKM and ADRO is 22.62%, 46.63% and 30.75%, respectively. If the investor chooses minimum risk, the combined allocation of BBRI, TLKM and ADRO shares is 34.73%, 51.21% and 14.06%, respectively.

Keywords: Stochastic process, geometric Brownian motion, optimal portfolio

1. Introduction

Investment is a commitment of a certain amount of funds or other resources made in the present, with the aim of gaining profits in the future (Szegö, 2014). The reason someone undertakes stock investment activities is to make a profit. The profits obtained from investing in shares can be seen from how many share returns are generated by the company that has been given funds by the investor. However, share prices always experience changes or price movements every day, making it difficult for investors or someone who carries out stock investment activities to predict the expected share price. This causes uncertainty in the value of stock returns that can be generated. For this reason, a simulation or mathematical calculation of the share price movement is needed.

A stock price movement at a certain time is a stochastic process. Basically, the mathematical model of stock price movements is described in a Stochastic Differential Equation (PDS). From this PDS model a model will be formed, one of which is the geometric Brownian motion model. Geometric Brownian motion results in future stock prices always having a positive value and stock price returns having a normal distribution. This model can be used to predict stock prices in the future.

An investor can predict stock price movements with a geometric Brownian motion model. This model is used to reduce the risks that can occur in investments. When making an investment, an investor needs to manage a certain amount of funds that will be allocated to various stock options by looking at the movement of the stock price. Shares that are owned with the aim of obtaining a profit in the future are called a portfolio. The basis for forming a portfolio is how an investor allocates funds to various investment options that will generate profits for him in the future (return expectations) by minimizing the risk of the portfolio that has been formed. Investors face many possible portfolios that can be formed from the shares they already own. Determining the optimal portfolio is the portfolio with the best combination of expected return and risk (Goetzmann, 2014). The optimal portfolio can be done in several ways, one of which is the Markowitz model.

Research on geometric Brownian motion models and Markowitz models has been carried out previously. Research on geometric Brownian motion was carried out by Seru (2022). In their research, Seru (2022) discussed the application of geometric Brownian motion to Aneka Tambang Persero Tbk (IDX: ANTM) share price data, and concluded that ANTM share price data for the period 2 January 2019 to 30 December 2019 could be modeled following Brownian motion. geometry with an observation time every fifteen minutes with ANTM share prices tending to fluctuate. Another research was conducted by Yuwono (2017). In his research, the results showed that there were differences between the single index model and the Markowitz model. The difference can be seen from the portfolio return expectations, namely the Markowitz model has a higher portfolio return expectation value than the portfolio returns expectation value of the single index model.

Based on this description, the researcher is interested in conducting research with the title "Determining the Optimal Portfolio on Moving Stock Prices Using Geometric Brownian Motion". The geometric Brownian motion model that is formed will then be applied to the closing share prices of Bank Rakyat Indonesia Tbk (IDX: BBRI), Telekomunikasi Indonesia Tbk (IDX: TLKM) and Adaro Energy Tbk (IDX: ADRO) shares.

2. Method

The method used in this research is a case study. Daily closing stock price data calculated on weekdays (Monday to Friday, excluding holidays) from August 1 2021 to July 31 2022. This research also uses the help of computer programs, namely Microsoft Excel 2019 and R-Studio. The steps used in analyzing research data are as follows:

1. form a model of stock price movements;
2. describe daily closing price data for BBRI, TLKM and ADRO shares;
3. calculate BBRI, TLKM and ADRO stock returns;
4. carry out a normality test using the Kolmogorov-Smirnov test on return data from each stock;
5. Simulate moving stock prices using the geometric Brownian Motion model; And
6. form an optimal portfolio using the Markowitz model from data on the three shares.

3. Results and Discussion

3.1 Stock Price Model

The stock price movement model is divided into two parts, the first is the deterministic part which is denoted μdt, while the second part is a model of random (stochastic) stock price changes which is denoted $\sigma \Delta W_t$. The μ value is the rate of return while the σ value is stock volatility which is used to measure random fluctuations in stock prices. The values of $\mu$and σ can be estimated using previous stock prices. According to Campolieti (2018) model of stock price movements using a geometric Brownian motion model is:

$$S_t = S_0 \, e^{\left(\left(\mu t - \frac{1}{2}\sigma^2 t\right) + Z\sigma\sqrt{t}\right)},$$

With $Z = \dfrac{\ln\left(\frac{S_t}{S_0}\right) - \left(\mu t - \frac{1}{2}\sigma^2 t\right)}{\sigma\sqrt{t}}$.

The model above is then called the geometric Brownian motion model. This model presents stock prices in the future period as positive and returns with a normal distribution.

3.2 Data Description

The daily stock data used are shares of Bank Rakyat Indonesia Tbk (IDX: BBRI), Telekomunikasi Indonesia Tbk (IDX: TLKM) and Adaro Energy Tbk (IDX: ADRO) from August 2021 to July 2022 obtained from www.finance.yahoo. com. The following is a data description of the closing prices of BBRI, TLKM and ADRO shares in the Table 1.

Table 1: Description of Closing Share Price Data

Descriptive Statistics of Daily Closing Prices of BBRI Shares		Descriptive Statistics of TLKM Daily Closing Share Prices		Descriptive Statistics of ADRO Daily Closing Share Prices	
Mean	4230.3	*Mean*	4042.4	*Mean*	2313.5
Median	4220	*Median*	4110	*Median*	2240
Standard Deviation	306.62	*Standard Deviation*	388.42	*Standard Deviation*	682.87
Minimum	3491	*Minimum*	3230	*Minimum*	1245
Maximum	4940	*Maximum*	4770	*Maximum*	3650
N	243	N	243	N	243

3.3 Stock Returns

According to (Goetzmann, 2014), share price returns are the results obtained from an investment. Investors get profits from share ownership of a company in the form of capital gains and dividends. The following are the returns for each stock in Table 2.

Table 2: Stock Return Value

Date	BBRI Share Returns	TLKM Share Returns	ADRO Share Returns
02 August 2021	-	-	-
03 August 2021	0.002670229	0.006024115	-0.014706147
.	.	.	.
.	.	.	.
.	.	.	.
28 July 2022	0.009216655	-0.014051753	0
29 July 2022	0	-0.002361276	-0.009188426

3.4 Normality Test

1. Hypothesis Formulation

The Kolmogorov-Smirnov (K-S) test hypothesis can be stated as:

H_0: Bank Rakyat Indonesia Tbk (IDX: BBRI) share return data

Telekomunikasi Indonesia (IDX: TLKM) and Adaro Energy (IDX: ADRO) have normal distribution

H_1 Bank Rakyat Indonesia Tbk (IDX: BBRI) share return data Telekomunikasi Indonesia (IDX: TLKM) and Adaro Energy (IDX: ADRO) is not normally distributed.

2. Significance Level

The significance level used is $\alpha = 5\%$

3. Decision Making Criteria

Decisions are taken based on the $p - value$. If the $p - value < 0.05$ then H_0 is rejected and if the $p - value > 0.05$ then H_0 is accepted. The stock returns obtained by Bank Rakyat Indonesia Tbk (IDX: BBRI), Telekomunikasi Indonesia (IDX: TLKM) and Adaro Energy (IDX: ADRO) have the following p-values as Table 3:

Tabel 3. P-value Kolmogorov-Smirnov Test

Share Name	*P-value*
IDX: BBRI	0.11590
IDX: TLKM	0.05903
IDX: ADRO	0.06351

Because the $p - value$ of each stock is greater than α, H_0 is accepted.

4. Conclusion

Share returns of Bank Rakyat Indonesia Tbk (IDX: BBRI), Telekomunikasi Indonesia (IDX: TLKM) and Adaro Energy (IDX: ADRO) are normally distributed.

3.5 Stock Price Simulation

The geometric Brownian motion model obtained during model formation is an estimated value of stock price movements. Furthermore, this model can be used to simulate stock price movements. The following is a simulation of BBRI, TLKM and ADRO share price movements.

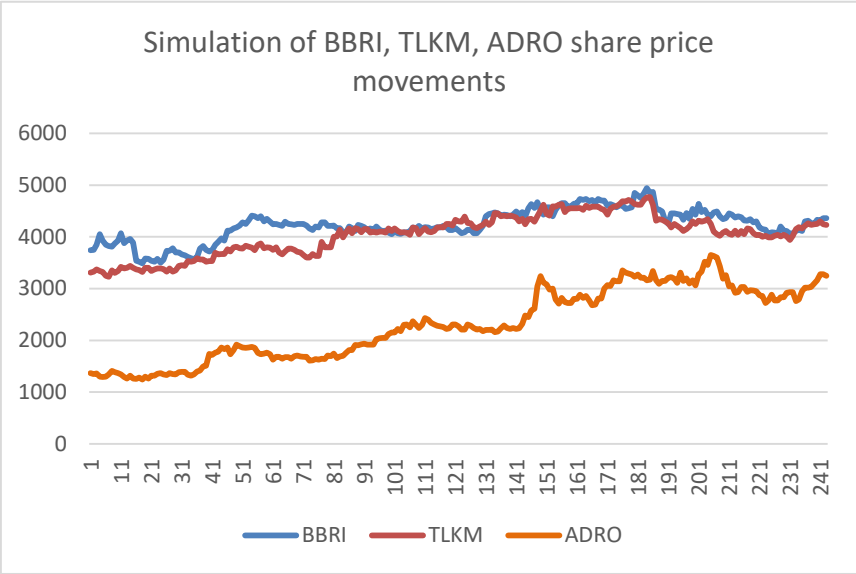

Figure 1: Simulation of Stock Price Movements Using the Geometric Brownian Motion Model

3.6 Optimal Portfolio

a. Return expectations and stock volatility

Expected stock returns are the sum of all stock returns which are then divided by the number of observation periods. Stock risk is a representation of stock volatility. This stock volatility calculation is used to determine the rise and fall (fluctuation) of a share price. The following are the expected return and stock volatility values as Table 4.

Table 4: Return Expectations and Stock Volatility

No.	Share Name	Return expectations	Stock volatility
1	IDX: BBRI	0.000633828	0.018553715
2	IDX: TLKM	0.001013445	0.016356699
3	IDX: ADRO	0.003569604	0.031355015

In Table 4, the expected stock returns for the three stocks are shown. The shares of Bank Rakyat Indonesia Tbk (IDX: BBRI), Telekomunikasi Indonesia (IDX: TLKM) and Adaro Energy (IDX: ADRO) have positive expected stock return values. This means that the stock tends to rise as time t increases.
In Table 4, it appears that the shares that have the greatest risk value are the IDX: ADRO shares. In other words, IDX: ADRO shares have increased more than other shares.

b. Correlation Coefficient Matrix

A stock correlation matrix is a matrix whose entries are coefficients from a group of stocks, with the diagonal entry being the number 1, while the other data is in the form of correlation coefficients between stocks. The following correlation coefficients presented in Table 5.

Table 5: Correlation Coefficient Matrix

Share Name	Correlation coefficient		
	IDX: BBRI	IDX: TLKM	IDX: ADRO
IDX: BBRI	1	0.2100732	0.1514581
IDX: TLKM	0.2100732	1	0.1554118
IDX: ADRO	0.1514581	0.1554118	1

Shows the relationship between shares. In Table 5 it can be seen that the correlation coefficient between shares is positive. This means that the price movement of the shares is unidirectional so that when the price of one share increases, the prices of other shares also tend to rise, and vice versa.

c. Matriks Variani-Kovarians

The variance-covariance matrix is a combination of the variance of each variable and the covariance of two different variables. The variance-covariance matrix has a diagonal whose elements are the variance of each variable while the other data is the covariance of two different variables. The following is the variance-covariance matrix presented in Table 6.

Table 6: Data Description

Share Name	Variance-Covariance		
	BBRI	TLKM	ADRO
BBRI	3.456687×10^{-4}	64.01703×10^{-4}	88.47665×10^{-4}
TLKM	64.01703×10^{-4}	2.686518×10^{-4}	80.03594×10^{-4}

ADRO	88.47665×10^{-4}	80.03594×10^{-4}	9.872164×10^{-4}

Based on Table 6, the covariance is positive, meaning that the values of the two stocks move in the same direction, that is, if one increases, the other will also increase and vice versa. The results of the variance and covariance values in Table 6 show that if IDX: BBRI, IDX: TLKM and IDX: ADRO shares are owned separately by each investor, then investors bear a higher risk than if they owned these shares together.

d. Proportion of Funds for Each Share

Forming an optimal portfolio first begins with looking for an efficient combination of portfolios. Based on calculations using the solver in Microsoft Excel 2019, the proportion of funds obtained can then be used to form an efficient portfolio combination as Table 7.

Table 7: Proportion of Funds for Each Share

Combination	BBRI	TLKM	ADRO	Amount
A	10.51%	42.05%	47.44%	100%
B	13.54%	43.19%	43.27%	100%
C	16.57%	44.34%	39.10%	100%
D	19.60%	45.48%	34.92%	100%
E	22.62%	46.63%	30.75%	100%
F	25.65%	47.77%	26.58%	100%
G	28.68%	48.92%	22.40%	100%
H	31.70%	50.06%	18.23%	100%
I	34.73%	51.21%	14.06%	100%

9 portfolio combinations will be formed as in Table 7. Next we will look for the portfolio expectations and portfolio risk of all the efficient portfolio combinations listed in Table 7.

e. Portfolio Expectations and Portfolio Risk

Next, the portfolio expectations and portfolio risks will be calculated from all the efficient portfolio combinations listed in Table 8. The portfolio expectations and portfolio risks are as Table 8.

Table 8: Portfolio Expectations and Portfolio Risk

Portfolio	Portfolio Expectations	Portfolio Risk
A	0.002186179	0.017880667
B	0.002068031	0.016995374
C	0.001949882	0.016174042
D	0.001831733	0.015426890
E	0.001713585	0.014765185
F	0.001595436	0.014200874
G	0.001477287	0.013745958
H	0.001359138	0.013411575
I	0.001240990	0.013206881

Table 8 displays the results of calculating portfolio expectations and portfolio risk. Furthermore, by applying Markowitz's portfolio theory, an optimal portfolio is obtained from the 9 efficient portfolios that have been formed. For an optimal portfolio, only one efficient portfolio combination is needed. The optimal portfolio chosen follows the investor's expected preferences based on the portfolio risk and expected return of a particular portfolio based on the Markowitz method. The following is a plot of 9 portfolio combinations (efficient frontier) from the Markowitz method shown in Figure 2.

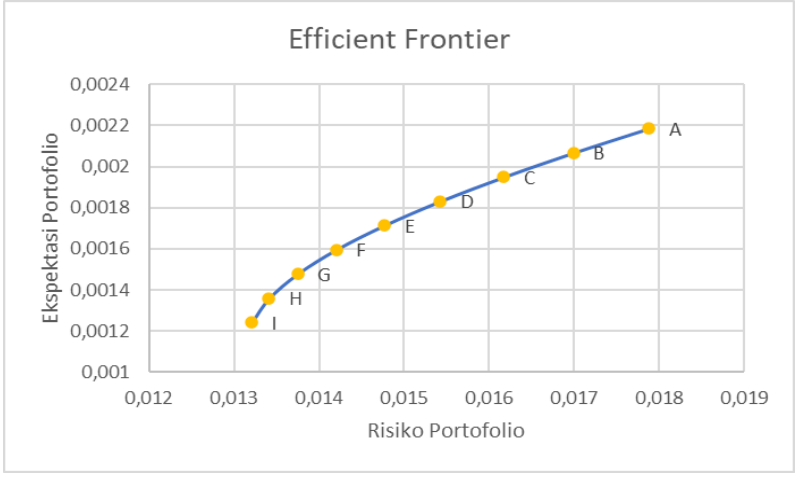

Figure 2: Mean-Variance Efficient Frontier Graph

From all the efficient portfolios formed, only one portfolio will be selected which will be used as the optimal portfolio with various preferences in accordance with the Markowitz model. If an investor takes a high risk with the expectation of a high expected return, then portfolio A is selected with a portfolio risk of 1.79%, a portfolio return rate of 0.21%, and has a portfolio combination for BBRI, TLKM, ADRO shares respectively. also amounted to 10.51%, 42.05%, and 47.44%.

On the other hand, if the investor takes the minimum risk then the portfolio with the smallest risk is selected with portfolio I being selected at 1.32%, the portfolio return rate is 0.12%, and has a portfolio combination for BBRI, TLKM, ADRO shares respectively. also amounted to 34.73%, 51.21%, and 14.06%. Meanwhile, if an investor is in the middle, which means taking risks that are in the middle, then the investor chooses portfolio E by taking a risk of 1.48%, a portfolio return rate of 0.17% which has a portfolio combination for BBRI, TLKM shares, ADRO respectively amounted to 22.62%, 46.63% and 30.75%.

4. Results and Discussion

Based on the results and discussion in this research, the following conclusions can be obtained:

1. The stock price movement model using the geometric Brownian motion model is

$$S_t = \mathrm{S}_0\, e^{\left(\left(\mu t - \frac{1}{2}\sigma^2 t\right) + Z\sigma\sqrt{t}\right)},$$

with $Z = \frac{\ln\left(\frac{S_t}{S_0}\right) - \left(\mu t - \frac{1}{2}\sigma^2 t\right)}{\sigma\sqrt{t}}$.

2. Based on optimal portfolio calculations using the Markowitz model, 3 portfolio combinations are obtained according to investor preferences. If investors like high risks to get high returns, the combined fund allocation for BBRI, TLKM and ADRO shares is 10.51%, 42.05% and 47.44% respectively. If investors do not like high risks but still want to get a return that is balanced with risk, the combined fund allocation for BBRI, TLKM and ADRO shares is 22.62%, 46.63% and 30.75% respectively. And if investors choose minimum risk, the combined fund allocation for BBRI, TLKM and ADRO shares will be 34.73%, 51.21% and 14.06%, respectively.

References

Goetzmann, W. N., Brown, S. J., Gruber, M. J., & Elton, E. J. (2014). Modern portfolio theory and investment analysis. *John Wiley & Sons*, *237*.

Yuwono, T., & Ramdhani, D. (2017). Comparison analysis of portfolio using Markowitz model and single index model: Case in Jakarta Islamic Index. *Journal of Multidisciplinary Academic*, *1*(1), 25-31.

Seru, F., Suhendra, C. D., & Saputro, A. D. (2022). Implementation of Geometric Brownian Motion to Predict Crude Oil Prices. *Numerical: Jurnal Matematika dan Pendidikan Matematika*, *6*(2), 141-152.

Campolieti, G., & Makarov, R. N. (2018). *Financial mathematics: a comprehensive treatment*. CRC Press.

Szegö, G. P. (2014). *Portfolio theory: with application to bank asset management*. Academic Press.