



## Optimal Portfolio Risk Analysis using the Monte Carlo Method

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### Abstract

Investment is an activity carried out with the expectation of gaining profits in the future through the management of investment assets. Investment assets can include buildings, gold, and stocks. Investment activities are inseparable from the concepts of return and risk. The relationship between the expected rate of return and the level of risk is linear. However, risk can be avoided or reduced through portfolio diversification. Evaluating investment risk is crucial for investors to determine which risky assets to choose. One popular method for assessing the risk of a portfolio is using Value at Risk (VaR). In VaR calculations, Monte Carlo is considered the most effective method. In this paper, a risk analysis of the optimal portfolio is conducted using the Monte Carlo method. The analyzed optimal portfolio consists of shares in BBKA, TLKM, BBRI, BBNI, BMRI, ADRO, GGRM, and UNTR. The results indicate that the potential loss for the investor is no more than IDR 705,634 with an initial fund of 1 billion.

**Keywords:** Risk analysis, Monte Carlo, Optimal Portfolio

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### 1. Introduction

Investment involves the efficient management of current financial wealth with the expectation of gaining profits in the future (Laopodis, 2020). Assets in investment are divided into financial assets and real assets (Fevurly, 2018). Real assets are tangible and measurable investment assets, such as land, buildings, and gold. On the other hand, financial assets are financial instruments traded in money markets, capital markets, and derivative markets (Astuti et al., 2020; Kholilurrahman & Alim, 2022).

One of the most popular instruments in the financial market is stocks, which are considered high-risk investments (Hendarto et al., 2021). Investment risk is generally categorized into systematic risk and unsystematic risk. Systematic risk is unavoidable and influenced by macroeconomic factors that affect the entire market, such as economic and political conditions. Unsystematic risk, on the other hand, can be mitigated through diversification, as it is specific to a particular company or industry. Diversification of stock investments can be achieved by forming a stock portfolio (Astuti et al., 2020).

A portfolio is a collection of assets owned by an individual or a company to obtain investment returns. The optimal portfolio is one with the best combination of expected return and risk. To achieve an optimal portfolio, one must first determine an efficient portfolio. An efficient portfolio is where the expected return is maximized for a given level of risk (Mahayani & Suarjaya, 2019).

Risk evaluation plays a crucial role in financial analysis related to substantial fund investments. The primary focus for any prospective investor is to determine which risky assets to acquire, closely related to the amount of funds to be invested (Astuti et al., 2020). One method to assess the risk of a portfolio is using Value at Risk (VaR). According to Jorion (2007), VaR has gained popularity as a widely used risk measurement instrument. There are three main approaches to calculating VaR: the parametric method (also known as the variance-covariance method), the historical simulation method, and the Monte Carlo simulation method. The Monte Carlo method is considered the most effective approach for measuring VaR, as it involves repeated simulations using random number generators to produce random values at certain probability frequencies. These simulation results are then used as estimates to forecast the future movement conditions of stocks (Jorion, 2007; Tardivo, 2002).

This paper focuses on the optimal portfolio risk analysis using the Monte Carlo method. The object used in this article is the optimal investment portfolio consisting of stocks BBKA, TKIM, BBRI, BBNI, BMRI, ADRO, GGRM, UNTR, as presented in the article titled "Portfolio Optimization of the Mean-Absolute Deviation Model of Some

Stocks using the Singular Covariance Matrix" (Kalfin et al., 2019). The objective is to obtain the results of risk analysis for the previously formed portfolio.

## 2. Literature Review

### 2.1. Portfolio

Portfolio represents a blend of assets, encompassing both tangible and financial investments held by an investor. (Astuti et al., 2020). The core principle behind portfolio construction is risk mitigation through diversification. This involves distributing an investor's funds across various investment options that exhibit negative correlations, aiming to yield optimal returns. Portfolio theory revolves around selecting the right mix of assets to maximize expected returns while managing the specific level of risk the investor is comfortable taking on.

According to Kalfin et al. (2019), the average vector is formed as follows:

$$\boldsymbol{\mu}^T = (\mu_1, \mu_2, \dots, \mu_N) \quad (1)$$

whereas the expected return of a stock is mathematically defined as follows:

$$\mu_i = E[R_i] = \int_{-\infty}^{\infty} (r_i f(r_i)) dr_i \quad (2)$$

From the estimation results of variance and covariance between shares, a variance-covariance matrix is formed, organized as follows:

$$\mathbf{M} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \dots & \sigma_{2N} \\ \vdots & \vdots & \dots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \dots & \sigma_N^2 \end{pmatrix} \quad (3)$$

whereas the estimated value of covariance between shares  $i$  and  $j$  is expressed as  $\sigma_{ij}$ , which mathematically defined as follows:

$$\sigma_{ij} = E[(R_i - \mu_i)(R_j - \mu_j)] = \rho_{ij}\sigma_i\sigma_j \quad (4)$$

with a stock weight vector

$$\mathbf{w}^T = (w_1, w_2, \dots, w_N) \quad (5)$$

### 2.2. Risk

Risk embodies the uncertainty affecting a company, presenting both positive and negative aspects. Addressing negative risks is crucial. When defining risk as the extent of deviation between achieved and desired outcomes, dispersion is employed as a measure (Lahi et al., 2023). Another approach to estimating the return with  $w_i$  states the proportion of funds invested in stock  $i$  is given as follows:

$$R_p = \sum_{i=1}^N w_i R_i \quad (6)$$

The average expected return of a portfolio and the square of the standard deviation per period based on equation (6) can be computed using the following formula as

$$\mu_p = E[R_p] = \sum_{i=1}^N w_i E[R_i] = \sum_{i=1}^N w_i \mu_i = \mathbf{w}^T \boldsymbol{\mu} \quad (7)$$

$$\sigma_p^2 = E[(R_p - E[R_p])^2] = \sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{ij} = \mathbf{w}^T \mathbf{M} \mathbf{w} \quad (8)$$

### 2.3. Portfolio Optimization based on Average Variance

Portfolio optimization involves crafting the most fitting blend of securities within the framework of expected returns, risk considerations, and investment constraints. According to Markowitz's theory, which cited in Hafize, M.D. and Umut U. (2013), optimal portfolios lie on an efficient frontier determined by the interplay of expected returns and risk perception. Portfolios situated along this efficient frontier curve represent the highest achievable returns at specific levels of risk.

The Portfolio Optimization model aims to identify a portfolio with the lowest variance (thus minimizing risk) while achieving a specified expected return level.

$$\max \left\{ 2\tau \mathbf{w}^T \boldsymbol{\mu} - (\mathbf{w}^T \mathbf{M} \mathbf{w})^{\frac{1}{2}} \right\} \\ \text{Obstacles } \mathbf{w}^T \mathbf{e} = 1 \quad (9)$$

Lagrange multiplier function of the equation (10) is given as follows:

$$L = 2\tau \mathbf{w}^T \boldsymbol{\mu} - (\mathbf{w}^T \mathbf{M} \mathbf{w})^{\frac{1}{2}} + \lambda (\mathbf{w}^T \mathbf{e} - 1) \quad (10)$$

$$\frac{\partial L}{\partial \mathbf{w}} = 2\tau \boldsymbol{\mu} - \frac{1}{2} 2 \frac{\mathbf{M} \mathbf{w}}{(\mathbf{w}^T \mathbf{M} \mathbf{w})^{\frac{1}{2}}} + \lambda \mathbf{e} \\ \frac{\mathbf{M} \mathbf{w}}{(\mathbf{w}^T \mathbf{M} \mathbf{w})^{1/2}} = 2\tau \boldsymbol{\mu} + \lambda \mathbf{e} \quad (11)$$

## 2.4. Value at Risk (VaR)

In 1994, J.P. Morgan introduced the Value at Risk (VaR) concept as a pivotal risk assessment tool. According to Jorion, as referenced in Penza and Bansal's work (2001), 'VaR encapsulates the expected maximum loss or worst-case loss over a target horizon within a specified confidence level.' According to Edwards (2014) as cited in Astuti et al. (2020), defines Value at Risk as 'the maximum anticipated loss on a financial instrument or a portfolio of financial instruments over a defined period and at a given confidence level.

These definitions underscore two vital factors in describing VaR. Firstly, VaR computation hinges on the time frame within which potential losses might occur—typically, a longer duration implies a higher VaR value. Secondly, VaR assessment necessitates a specified confidence level, indicating the probability of a loss exceeding the VaR figure. In the context of a portfolio, VaR represents an estimate of the maximum potential loss the portfolio might incur over a defined time span at a certain confidence level. Hence, the actual loss experienced by the portfolio during ownership may fall below the VaR-established threshold.

According to Lahi et. al. (2023), if  $W_0$  stands as the initial investment asset model, the asset's value at the end of the time period can be expressed as:

$$W = W_0(1 + R) \quad (12)$$

If the minimum asset value within the confidence interval  $(1 - \alpha)$  is:

$$W^* = W_0(1 + R^*) \quad (13)$$

Then the Value at Risk (VaR) within the confidence interval  $(1 - \alpha)$  can be expressed as:

$$VaR_{(1-\alpha)} = W_0 R^* \quad (14)$$

with  $R^*$  being negative, it can be formulated as follows.

$$R^* = E(R) - Z_{1-\alpha} \sqrt{\text{Var}(R)} \quad (15)$$

The expected return escalates linearly concerning time ( $t$ ), while the standard deviation increases proportionally with the square root of time. Therefore, the VaR value can be defined as follows.

$$E(R) = \mu(t) = \mu t \quad (16)$$

$$\text{Var}(R) = \sigma^2(t) = \sigma^2 t \Rightarrow \sqrt{\text{Var}(R)} = \sigma(t) = \sigma \sqrt{t} \quad (17)$$

Hence the equation (6) can be formulated as follows.

$$R^* = \mu t - Z_{1-\alpha}(t) \sigma \sqrt{t} \quad (18)$$

Further, the computation of VaR at a confidence level of  $(1 - \alpha)$  after  $t$  periods can be expressed through equation (7)

$$VaR_{1-\alpha}(t) = W_0 R^* \sqrt{t} \quad (19)$$

where,

$VaR_{1-\alpha}(t)$  : VaR at a confidence level of  $(1 - \alpha)$  after  $t$  periods

$W_0$  : initial investment in assets

$R^*$  :  $\alpha$ -th quantile of the return distribution

$t$  : the period for holding stocks.

Furthermore, as per Astuti et al. (2020), the equation (8) below delineates the method for computing VaR as well.

$$VaR = Mean - \sigma \quad (7)$$

## 2.5. Monte Carlo

The application of the Monte Carlo simulation method for risk measurement was introduced by Boyle in 1977. Estimating Value at Risk (VaR) using Monte Carlo simulation essentially involves simulating random numbers based on the data's characteristics, which are then utilized to estimate the VaR value. For VaR measurement using the Monte Carlo method, adherence of the data to a normal distribution is essential (Astuti et al., 2020).

## 3. Materials and Methods

### 3.1. Materials

The stock data was obtained from <https://finance.yahoo.com>, which had been previously researched by Kalfin et al (2019) for the period from October 2015 to October 2018. The data used consisted of 8 (eight) shares in the form of BBKA, TKIM, BBNI, BBRI, BMRI, ADRO, GGRM and UNTR.

### 3.2. Methods

The model used in stock portfolio optimization is the mean-standard deviation, using a singular covariance matrix, and risk analysis using the Monte Carlo method. Discussions include stock returns and mean-standard deviation models.

## 4. Results and Discussion

The portfolio used in this study is an optimal portfolio taken from a journal paper by Kalfin *et al.* (2019) entitled Portfolio Optimization of the Mean-Absolute Deviation Model of Some Stocks using the Singular Covariance Matrix. The following is the optimal portfolio table that will be used in Table 1:

**Table 1: Optimal Portfolio Table**

Weights							
BBKA	TKIM	BBRI	BBNI	BMRI	ADRO	GGRM	UNTR
0.172189	0.266369	0.099619	0.099619	0.086965	0.037505	0.065172	0.172563

Portfolio risk is calculated using Value at Risk with the Monte Carlo simulation method carried out with the help of the Risk Analysis Simulator program on Microsoft Excel. The following are the results of simulation calculations that contain the mean, standard deviation, variance, risk, Z-stat, and confidence level can see in Table 2:

**Table 2: Value at Risk Portofolio Table**

Statistics	Value
Trial	500
Mean	0.001844
Standard Deviation	0.00069
Variance	0.000706
Risk	0.38266
Z-stat	1.65
Confidence Level	95%

Based on the results of data simulations that have been carried out, the Value at Risk of a portfolio can be calculated using the equation formula (7) so that the following calculation is obtained:

$$VaR = 0.001844 - 1.65 (0.00069) = 0.000706$$

Suppose the initial investment is 1 billion, the following calculation is obtained:

$$VaR (IDR) = 0.000706 \times 1,000,000,000 = IDR 705,634$$

Based on the calculations that have been carried out, the resulting VaR value is 0.000706 with a confidence level of 95%. Thus, it can be interpreted that if the initial funds invested in the portfolio are IDR 1,000,000,000, there is a 95% confidence that the loss that investors may suffer will not be more than IDR 705,634.

## 5. Conclusion

In this research, Value at Risk with Monte Carlo simulation is used to calculate the risk level in an optimal portfolio consisting of BBKA, TLKM, BBRI, BBNI, BMRI, ADRO, GGRM, and UNTR stocks. Based on this research, it can be concluded that the optimal portfolio has a VaR value of 0.000706, which indicates that the loss that will be suffered by investors who use this portfolio will not be more than IDR 705, 634 with initial funds of 1 billion.

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