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Stationary Distribution Markov Chain for COVID-19 Pandemic

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Abstract

Coronavirus disease (COVID-19) is a new disease found in the late 2019. The first case was reported on December 31, 2019 in Wuhan, China and spreading all over the countries. The disease was quickly spread to all over the countries. There are 206.900 cases confirmed by March 18, 2020 causing 8.272 death. It was predicted that the number of confirmed cases will continue to increase. On January 30, 2020, WHO declared this as pandemic for the 6th time ever since the swine influenza. There are a lot of researchers which discuss pandemic spreading caused by virus with mathematical modelling. In this paper, we discuss a long-term prediction over the COVID-19 spreading using stationary distribution markov chain. The goal is to analyze the prediction of infected people in long-term by analyzing the COVID-19 daily cases in an observation interval. By analyzing the daily cases of COVID-19 in Indonesia from March 2nd, 2020 until November 1st, 2020, result shown that 53.91% of probability that the COVID-19 daily case will incline in long-term, 44.86% of chance will decline, and 1.23% of chance will stagnant.

Keywords: COVID-19, Stochastic Processes, Stationary Distribution Markov Chain

1. Introduction

Coronavirus disease, or also known as COVID-19, is a new disease found in the late 2019. It caused by a virus called SARS-CoV-2 (Severe Acute Respiratory Syndrome Coronavirus 2). The first case of this disease was reported on December 31, 2019 in Wuhan, China and spreading all over the countries. It has some common symptoms i.e., fever, cough, and severe cases. In some cases, it can cause pneumonia, severe acute respiratory syndrome, kidney failure, or even death.

The disease was quickly spread to all over the countries. There are 206.900 cases confirmed by March 18, 2020 causing 8.272 death. It was predicted that the number of confirmed cases will continue to increase. On January 30, 2020, WHO declared this as pandemic for the 6th time ever since the swine influenza.

There are a lot of researches which discuss pandemic spreading caused by virus with mathematical modelling. Aldila et al (2018). discussed a SIR model for MERS-CoV Yong (2007) discussed a model about HIV. Sianturi (2015) used stochastic processes to analyze the spread of dengue hemorrhagic fever virus. In this paper, we discuss a long-term prediction over the COVID-19 spreading using stationary distribution markov chain. The goal is to analyze the prediction of infected people in long-term.

2. Materials and Methods

2.1. Discrete-Time Markov Chain

Let $\{X(n), n = 0,1,2,...\}$ be a discrete-time stochastic process with time parameter $n = 0,1,2,...$ and state space $i = 0,1,2,....$ In other words, $X_n = i$ define that the process is in state i at time . If the probability in the future time $(n+1)$ in a state j is dependent only to the the present condition in state i at the present time n, then the process is called a discrete-time Markov chain. Such probability is notated by p_{ij}, which define the transition probability from state i to state j. Later, we called p_{ij} as a transition probability (Osaki , 1992).

Later, we collect all the transition probability p_{ij} from every possible state i and j as a matrix **P** as discussed by [6] which formulated by:

$$\mathbf{P} = [p_{ij}] = \begin{bmatrix} p_{00} & p_{01} & p_{02} & \cdots \\ p_{10} & p_{11} & p_{12} & \cdots \\ p_{20} & p_{21} & p_{22} & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}$$

where $p_{ij} \geq 0$ and $\sum_{j=0}^{\infty} p_{ij} = 1, \forall i,j = 0,1,2,....$

If the transition probability in the future is independent from the present, then the process is called stationary distribution Markov chain.

2.2. State Classification

There are several state classification that we used in the terms of stationary distribution Markov chain i.e., irreducible, aperiodic and positive recurrent.

A process in Markov chain is called irreducible if and only if the process only has 1 communication class. In other words, all the state in the process are connected so that they can transits from any state to every other state in any possible step.

A state i is called aperiodic if and only if $d(i) = 1$, where $d(i)$ is formulated by

$$d(i) = \gcd\{n | n \geq 1, p_{ii}^n > 0\}$$

In other words, $d(i)$ is the great common divisor of all possible n which makes the process in state i can going back to the same state i by n step.

A state is i called recurrent if and only if $\sum_{n=1}^{\infty} p_{ii}^n = \infty$ (Osaki , 1992). Later, a state i is called positive recurrent if $\mu_i < \infty$, where μ_i represents the average recurrent time of state i.

Theorem 1. If any given state i is recurrent and aperiodic, then

$$\lim_{n\to\infty} p_{ii}^n = \frac{1}{\mu_i}$$

Theorem 2. If an irreducible Markov chain is positive recurrent and aperiodic, there exist the limiting probability π_j

$$\lim_{n\to\infty} p_{ij}^n = \pi_j > 0\,, \qquad (i, j = 0,1,2,\ldots)$$

which is independent to initial state i, where $(\pi_j, j = 0,1,2,\ldots)$ is a unique and positive solution to

$$\pi_j = \sum_{i=0}^{\infty} \pi_i p_{ij}\,, \qquad (j = 0,1,2,\ldots)$$

$$\sum_{j=0}^{\infty} \pi_j = 1$$

Later, π_j is called stationary distribution for state j (Osaki , 1992). The stationary distribution tells us the probability tendencies in long-time so it's not dependent to the initial state.

3. Results and Discussion

We analyzed the daily cases of COVID-19 data in Indonesia from March 2nd, 2020 until November 1st, 2020 which obtained from Indonesian COVID-19 Response Acceleration Task Force (Gugus Tugas Percepatan Penanganan COVID-19 Republik Indonesia) and can be accessed via https://bnpb-inacovid19.hub.arcgis.com/ (World Health Organization).

3.1 Results

In this paper, we define 3 kinds of state in state space i.e., the daily cases of COVID-19 is declining from the previous day which notated by 0, the daily cases of COVID-19 is the same from the previous day which notated by 1, and the daily cases of COVID-19 is inclining from the previous day which notated by 2.

After we analyzed the data, the number of each state occured are:

n_0:the number of state 0 occured $= 110$

n_1:the number of state 1 occured $= 3$

n_2:the number of state 2 occured $= 131$

Also, the transition probability p_{ij} from the data are:

$0 \to 0 = 38$	$1 \to 0 = 1$	$2 \to 0 = 70$
$0 \to 1 = 2$	$1 \to 1 = 1$	$2 \to 1 = 0$
$0 \to 2 = 69$	$1 \to 2 = 1$	$2 \to 2 = 61$

$p_{00} = \frac{38}{109}$ $p_{10} = \frac{1}{3}$ $p_{20} = \frac{70}{131}$

$p_{01} = \frac{2}{109}$ $p_{11} = \frac{1}{3}$ $p_{21} = 0$

$p_{02} = \frac{69}{109}$ $p_{12} = \frac{1}{3}$ $p_{22} = \frac{61}{131}$

So we get the transition probability matrix **P** below:

$$\mathbf{P} = \begin{bmatrix} \frac{38}{109} & \frac{2}{109} & \frac{69}{109} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{70}{131} & 0 & \frac{61}{131} \end{bmatrix}$$

By analyzing the state classification of this Markov chain, the result is that the Markov chain is irreducible, aperiodic, and positive recurrent. So, we can continued to analyze the stationary distribution of Markov chain by using Theorem 2 (Lingjie, 2020).

By using Theorem 2, we get the stationary distribution Markov chain for daily cases of COVID-19 is:

$$\pi = [\pi_0, \pi_1, \pi_2] = \left[\frac{109}{243}, \frac{1}{81}, \frac{131}{243}\right]$$

The result above means that based on the daily confirmed case from December 31st 2019 until April 16th 2020, the long-term probability of COVID-19 daily cases will decline is $\frac{109}{243} \approx 44.86\%$. The long-term probability of COVID-19 daily cases will stagnant is $\frac{1}{81} \approx 1.23\%$. The long-term probability of COVID-19 daily cases will incline is $\frac{131}{243} \approx 53.91\%$. This result depends on the observation data. In other words, the result may vary depends on the observation interval which can make different pattern of COVID-19 daily cases.

4. Conclussion

In this paper, we analyzed the daily cases of COVID-19 in Indonesia from March 2nd, 2020 until November 1st, 2020 with stationary distribution Markov chain to predict the long-term probability tendencies of the daily cases. By analyzing the COVID-19 daily cases from March 2nd 2020 until November 1st 2020, we can conclude that the long-term prediction of COVID-19 daily cases will decline is 44.86%. The long-term of COVID-19 daily cases will stagnant is 1.23%. The long-term of COVID-19 daily cases will incline is 53.91%.

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