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Optimal Forestry Model Control with Logging and Tourism Factors

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Abstract

Forest is one of the natural resources that need to be preserved because it has vital functions for humans both ecologically and economically. In this study, a mathematical model of forestry dynamics was developed by dividing the forest area, indigenous people, non-indigenous people, population pressure and economic incentives. The model was analyzed by dynamic system theory, the existence of equilibrium points and their stability were determined. Using the second Lyapunov method, global stability was also determined. In that forestry model, logging and tourism factors were added which affect the dynamics of forest biomass. The Pontryagin maximum principle was used to obtain optimal conditions from the model. Numerical simulation shows that the use of forests by logging and tourism, reduces the amount of forest biomass, but the forest remains sustainable. Utilization of forests bycontrols will maximize the benefits of logging and tourism in the associated forests.

Keywords: Forestry, Logging, Tourism, Stability Analysis, Optimal Control

1. Introduction

Forests are one of the most important parts of the ecosystem on Earth. Forests play a large role in preventing soil erosion, global warming, landslide, flooding and so on. Unfortunately, in Indonesia, the large clearing of land for oil palm plantations, mining, and forest over the land have caused deforestation in Indonesia. (Austin *et al.*, 2019). Human needs against forest resources tend to exploit primarily by non-indigenous peoples (Reyes-Garcia *et al.*, 2012). Indigenous peoples are the first inhabitants to settle an area. They are usually called indigenous peoples when they maintain a tradition of conformity with the area. Indigenous peoples tend to keep their area from excessive exploitation (Corrigan *et al.*, 2018; Mulyoutami *et al.*, 2009; Wadley & Coffer, 2004). It is important to involve the indigenous peoples in the effort of forest Preservation (Fatem *et al.*, 2018; Clay *et al.*, 2000; Sahide *et al.*, 2016). The tradition also attracts foreign residents or foreign communities because they reliance with nature. So, it attracts tourists to visit the area. It can be an economic potential within the region.

Researchers have used mathematical models about the dynamics of forest resources. In 1989, Shukla *et al.* (1989) proposed a mathematical model on forest resources. Agarwal *et al.* (2010) and Chaundhary *et al.* (2015) examined the influence of industrial pressure on the lowering of forest resources. Mirsa *et *al.* (2014) and Lata *et al.* (2018) showed that population pressure affects forest biomass. Mirsa & Lata (2013) also examined the technological efforts of the vision of forest biomass. In his research Dhar (2008), the mathematical model for forest resources was divided into two. While the influence of tourism on the Environment, Lacitignola (2007) stated that tourism affects forest quality.

From the above studies, models on the influence of population distribution, indigenous people and non-indigenous people, as well as the influence of forest utilization in the form of logging and tourism, have not been studied. So in this research will be discussed on the model of forestry mathematics that is managed by the indigenous peoples and non-indigenous peoples to its utilization in the form of logging and tourism.

2. Materials and Methods

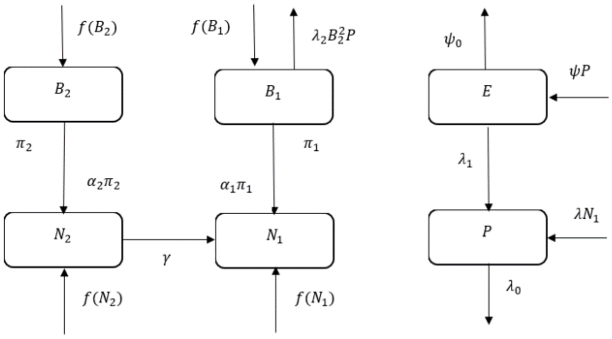

Figure 1: Forest Model Schematic Diagram

The main model used in this research is the forestry model with population pressure developed by Misra *et al.* (2014). The model in Misra *et al.* (2014) has been extended in Pratama *et al.* (2020) by dividing the utilization of the forest by two different level of exploitation. Pratama *et al.* (2020) showed the dynamics of the modified model without considering optimal exploitation. In this paper we add optimal control in the level of exploitation into the original model of Misra *et al.* (2014) as follows. In this article the original model of Misra *et al* (2014) is developed by dividing the human population into two parts, namely indigenous peoples (N_2) and non-indigenous peoples (N_1). The forest is also divided into two parts, a forest managed by non-indigenous peoples (B_1) and forests managed by the indigenous peoples(B_2). Both population and forest grow in the logistics growth. Indigenous peoples as well as non-indigenous peoples each make use of forests for their lives. However, what distinguishes is the non-indigenous population giving population pressure. Population pressure (P) is the effect of the existence of non-indigenous peoples who manage forests unsustainably. Thereby reducing carrying capacity of the forest they manage. To reduce population pressure, economic incentives (E) are given.

This Model has been represented by the schematic diagram in Figure 1 with the parameter description displayed in Table 1. Differential equations for the forestry model are as follows:

$$\frac{dB_1}{dt} = sB_1\left(1-\frac{B_1}{L_1}\right) - \alpha_1 B_1 N_1 - \lambda_2 B_1^2 P$$
$$\frac{dB_2}{dt} = sB_2\left(1-\frac{B_2}{L_2}\right) - \alpha_2 B_2 N_2$$
$$\frac{dN_1}{dt} = rN_1\left(1-\frac{N_1}{K_1}\right) + \pi_1\alpha_1 B_1 N_1 + \gamma N_1 N_2$$

$$\frac{dN_2}{dt} = rN_2\left(1-\frac{N_2}{K_2}\right)+\pi_2\alpha_2B_2N_2-\gamma N_1N_2$$
$$\frac{dP}{dt} = \lambda N_1-\lambda_0P-\lambda_1PE \quad (1)$$
$$\frac{dE}{dt} = \psi P-\psi_0E$$

By defining the parameters in table 1 below:

Table 1: Model parameters

Parameter	Description	Parameter	Description
s	Growth rate of forest biomass	r	Population growth rate
L_1	Carrying Capacity forest B_1	K_1	Carrying capacity of population N_1
L_2	Carrying capacity forest B_2	K_2	Carrying capacity of population N_2
α_1	Rate of reduction B_1 due to direct utilization by population N_1	α_2	Rate of reduction due to direct utilization by population B_1N_1
π_1	Rate of addition N_1 due to direct utilization of forest biomassB_1	π_2	Rate of addition N_2 due to direct utilization of forest biomass B_2
λ	Growth rate P	λ_0	Natural reduction rateP
λ_2	Carrying capacity B_1 reduction rate due to P	λ_1	Rate reduction P due E
γ	Urbanisation rate	ψ	Growth rate E
ψ_0	Natural reduction rate E		

Lemma 1 : Suppose the initial value of the system (1) is: $B_1(0)\geq 0, B_2(0)\geq 0, N_1(0)\geq 0, N_2(0)\geq 0, P(0)\geq 0 \quad E(0)\geq 0$. Then the solution of the System (1) is in the region $\Omega=\left\{(B_1,B_2,N_1,N_2,P,E): 0\leq B_1\leq L_1; 0\leq B_2\leq L_2; 0\leq N_1\leq N_{m1}; 0\leq N_2\leq N_{m2}; 0\leq P\leq \frac{\lambda}{\lambda_0}N_{m1}; 0\leq E\leq \frac{\lambda\kappa\psi}{\lambda_0\psi_0}N_{m1}\right\}$ with value $N_{m1}=\frac{K_1}{r}(r+\pi_1\alpha_1L_1+\gamma N_{m2})$ dan $N_{m2}=\frac{K_2}{r}(r+\pi_2\alpha_2L_2)$.

3. Results and Discussion

3.1 Steady states and their stability analysis

A fixed point or equilibrium point is obtained when each population growth rate on a system (1) is zero value. A condition of equilibrium in which the number of populations does not increase or decrease. Meanwhile, a fixed point stability determination can be discovered by entering the value into the Jacobian matrix. Then sought the Eigen value of the Jacobian matrix. Here is the fixed point and its stability:

Table 2: Types of stability for each fixed point

Fixed point value	Eigen value (Λ)	Type of stability
$B_1=0, B_2=0, N_1=0$ $N_2=0, P=0, E=0$	$\Lambda_1=-\psi, \Lambda_2=-\lambda_0, \Lambda_3=s$ $\Lambda_4=s, \Lambda_5=r, \Lambda_6=r$	Unstable (saddle point)
$B_1=0, B_2=0, N_1=0$ $N_2=K_2, P=0, E=0$	$\Lambda_1=s, \Lambda_2=-K_2\alpha_2+s,$ $\Lambda_3=K_2\gamma+r, \Lambda_4=-r,$ $\Lambda_5=-\lambda_0, \Lambda_6=-\psi_0$	Unstable (saddle point)

$B_1 = L_1, B_2 = 0, N_1 = 0$ $N_2 = 0, P = 0, E = 0$	$\Lambda_1 = s, \Lambda_2 = L_1\pi_1\alpha_1 + r,$ $\Lambda_3 = r, \Lambda_4 = -s\,,$ $\Lambda_5 = -\lambda_0, \Lambda_6 = -\psi_0$	Unstable (saddle point)
$B_1 = L_1, B_2 = 0, N_1 = 0$ $N_2 = K_2, P = 0, E = 0$	$\Lambda_1 = -r, \Lambda_2 = -K_2\alpha_2 + s,$ $\Lambda_3 = L_1\pi_1\alpha_1 + K_2\gamma + r,$ $\Lambda_4 = -s\,, \Lambda_5 = -\lambda_0,$ $\Lambda_6 = -\psi_0$	Unstable (saddle point)
$B_1 = 0, B_2 = L_2, N_1 = 0$ $N_2 = 0, P = 0, E = 0$	$\Lambda_1 = s, \Lambda_2 = r,$ $\Lambda_3 = L_2\pi_2\alpha_2 + r, \Lambda_4 = -s\,,$ $\Lambda_5 = -\lambda_0, \Lambda_6 = -\psi_0$	Unstable (saddle point)
$B_1 = L_1, B_2 = L_2, N_1 = 0$ $N_2 = 0, P = 0, E = 0$	$\Lambda_1 = L_1\pi_1\alpha_1 + r, \Lambda_2 = -s,$ $\Lambda_3 = L_2\pi_2\alpha_2 + r, \Lambda_4 = -s\,,$ $\Lambda_5 = -\lambda_0, \Lambda_6 = -\psi_0$	Unstable (saddle point)
$B_1 = 0, N_1 = 0$ $B_2 = -\dfrac{L_2 r(K_2\alpha_2 - s)}{K_2L_2\pi_2\alpha_2^2 + rs},$ $N_2 = \dfrac{K_2 s(L_2\pi_2\alpha_2 + r)}{K_2L_2\pi_2\alpha_2^2 + rs},$ $P = 0, E = 0$	$\Lambda_1 = s, \Lambda_2 = -\psi_0, \Lambda_3 = -\lambda_0$ $\Lambda_4 = \Lambda^{+*}, \Lambda_5 = \Lambda^{+*}, \Lambda_6 = \Lambda^{-*}$	Unstable (saddle point)
$B_1 = L_1, N_1 = 0$ $B_2 = -\dfrac{L_2 r(K_2\alpha_2 - s)}{K_2L_2\pi_2\alpha_2^2 + rs},$ $N_2 = \dfrac{K_2 s(L_2\pi_2\alpha_2 + r)}{K_2L_2\pi_2\alpha_2^2 + rs},$ $P = 0, E = 0$	$\Lambda_1 = -s, \Lambda_2 = -\psi_0, \Lambda_3 = -\lambda_0$ $\Lambda_4 = \Lambda^{+*}, \Lambda_5 = \Lambda^{-*}, \Lambda_6 = \Lambda^{-*}$	Unstable (saddle point)
$B_1 = 0, B_2 = 0, N_1 = K_1$ $N_2 = 0, P = P^1, E = E^1$	$\Lambda_1 = s - \alpha_1K_1, \Lambda_2 = s, \Lambda_3 = r$ $\Lambda_4 = \lambda_0, \Lambda_5 = \Lambda^{-\#}, \Lambda_6 = \Lambda^{-\#}$	Unstable
$B_1 = 0, B_2 = L_2, N_1 = K_1$ $N_2 = 0, P = P^1, E = E^1$	$\Lambda_1 = s - \alpha_1K_1, \Lambda_2 = -s,$ $\Lambda_3 = r$ $\Lambda_4 = \lambda_0, \Lambda_5 = \Lambda^{-\#}, \Lambda_6 = \Lambda^{-\#}$	Unstable
$B_1 = 0, B_2 = 0, N_1 = N_1^1$ $N_2 = N_2^1, P = P^2, E = E^2$	$\Lambda_1 = \Lambda^{-*}, \Lambda_2 = \Lambda^{-*}, \Lambda_3 = \Lambda^{+*}$ $\Lambda_4 = \Lambda^{-*}\ \Lambda_5 = \Lambda^{-\#}, \Lambda_6 = \Lambda^{-\#}$	Unstable
$B_1 = 0, B_2 = B_2^1, N_1 = N_1^2$ $N_2 = N_2^2, P = P^3, E = E^3$	$\Lambda_1 = \Lambda^{-*}, \Lambda_2 = \Lambda^{-*}, \Lambda_3 = \Lambda^{+*}$ $\Lambda_4 = \Lambda^{-*}\ \Lambda_5 = \Lambda^{-\#}, \Lambda_6 = \Lambda^{-\#}$	Unstable
$B_1 = B_1^1, B_2 = 0, N_1 = N_1^3$ $N_2 = 0, P = P^4, E = E^4$	$\Lambda_1 = \Lambda^{-*}, \Lambda_2 = \Lambda^{-*}, \Lambda_3 = \Lambda^{+*}$ $\Lambda_4 = \Lambda^{+*}\ \Lambda_5 = \Lambda^{-\#}, \Lambda_6 = \Lambda^{-\#}$	Unstable
$B_1 = B_1^1, B_2 = L_2, N_1 = N_1^3$ $N_2 = 0, P = P^4, E = E^4$	$\Lambda_1 = \Lambda^{-*}, \Lambda_2 = \Lambda^{-*}, \Lambda_3 = \Lambda^{+*}$ $\Lambda_4 = \Lambda^{-*}\ \Lambda_5 = \Lambda^{-\#}, \Lambda_6 = \Lambda^{-\#}$	Unstable
$B_1 = B_1^2, B_2 = 0, N_1 = N_1^4$ $N_2 = N_2^3, P = P^5, E = E^5$	$\Lambda_1 = \Lambda^{-*}, \Lambda_2 = \Lambda^{-*}, \Lambda_3 = \Lambda^{+*}$ $\Lambda_4 = \Lambda^{-*}\ \Lambda_5 = \Lambda^{-\#}, \Lambda_6 = \Lambda^{-\#}$	Unstable
$B_1 = B_1^*, B_2 = B_2^*, N_1 = N_1^*$ $N_2 = N_2^*, P = P^*, E = E^*$	$\Lambda_1 = \Lambda^{-*}, \Lambda_2 = \Lambda^{-*}, \Lambda_3 = \Lambda^{-*}$ $\Lambda_4 = \Lambda^{-*}\ \Lambda_5 = \Lambda^{-\#}, \Lambda_6 = \Lambda^{-\#}$	Stable

Table 2 shows that a stable point locally is only at a point $F^*(B_1^*, B_2^*, N_1^*, N_2^*, P^*, E^*)$ While the other point is unstable. It will be searched for whether the point F^* has global stability.

Theorems 1 The point $F^*(B_1^*, B_2^*, N_1^*, N_2^*, P^*, E^*)$ has global stability if it meets the following equalities:

$$\lambda_2^2 B_1^{*2} < 2\left(\frac{s}{L_1} + \lambda_2 P^*\right)\left(\frac{\kappa\psi(\lambda_0 + \lambda_1 E^*)}{\lambda_1 P^*}\right)$$
$$\frac{\kappa\psi\lambda^2}{\lambda_1 P^*} < \frac{2r(\lambda_0 + \lambda_1 E^*)}{K_1\pi_1} \quad (2)$$

3.2 Control Optimal Model

This section is discussed on the forestry model that has been given control. In this article, the control variables that are added to the model (1) are described as M_1 and M_2.The control variable M_1 represents the logging factor against the forest managed by the non-indigenous population. While the control variable M_2 represents the tourism factor of the forest managed by the indigenous peoples. It is assumed the logging factor and tourism reduced the amount of biomass from each forest. This reduction rate is represented with q_1 and q_2. So the model (1) becomes as follows:

$$\begin{aligned}
\frac{dB_1}{dt} &= sB_1\left(1 - \frac{B_1}{L_1}\right) - \alpha_1 B_1 N_1 - \lambda_2 B_1^2 P - q_1 B_1 M_1 \\
\frac{dB_2}{dt} &= sB_2\left(1 - \frac{B_2}{L_2}\right) - \alpha_2 B_2 N_2 - q_2 B_2 M_2 \\
\frac{dN_1}{dt} &= rN_1\left(1 - \frac{N_1}{K_1}\right) + \pi_1\alpha_1 B_1 N_1 + \gamma N_1 N_2 \\
\frac{dN_2}{dt} &= rN_2\left(1 - \frac{N_2}{K_2}\right) + \pi_2\alpha_2 B_2 N_2 - \gamma N_1 N_2 \\
\frac{dP}{dt} &= \lambda N_1 - \lambda_0 P - \lambda_1 PE \\
\frac{dE}{dt} &= \psi P - \psi_0 E
\end{aligned} \quad (3)$$

The main objective of this optimal control problem is to maximize the following objective functions

$$Max\, J(M_1, M_2) = \int_0^{tf} e^{-\delta t}\big((p_1 - v_1 q_1 M_1 B_1)q_1 M_1 B_1 + (p_2 - v_2 q_2 M_2 B_2)q_2 M_2 B_2 - c_1 M_1 - c_2 M_2\big) \quad (4)$$

where variable v_1 and v_2 are economics factors and δ is discount rate. q_1, q_2, c_1, c_2 are costs incurred for logging, tourism and economic incentives. Then, the variable control M_1^*, M_2^* will be searched so that

$$J(M_1^*, M_2^*) = \max_M\{J(M_1, M_2)\} \quad (5)$$

where

$$M = \{(M_1, M_2): [0, t_f] \to [0,1]\} \quad (6)$$

The Hamiltonian equation for this optimal control problem is

$$\begin{aligned}
H = {} & e^{-\delta t}\big((p_1 - v_1 q_1 M_1 B_1)q_1 M_1 B_1 + (p_2 - v_2 q_2 M_2 B_2)q_2 M_2 B_2 \\
& -c_1 M_1 - c_2 M_2\big) \\
& +\mu_1\left(sB_1\left(1 - \frac{B_1}{L_1}\right) - \alpha_1 B_1 N_1 - \lambda_2 B_1^2 P - q_1 B_1 M_1\right) \\
& +\mu_2\left(sB_2\left(1 - \frac{B_2}{L_2}\right) - \alpha_2 B_2 N_2 - q_2 B_2 M_2\right)
\end{aligned}$$

$$+\mu_3\left(rN_1\left(1-\frac{N_1}{K_1}\right)+\pi_1\alpha_1B_1N_1+\gamma N_1N_2\right)$$
$$+\mu_4\left(rN_2\left(1-\frac{N_2}{K_2}\right)+\pi_2\alpha_2B_2N_2-\gamma N_1N_2\right) \tag{7}$$
$$+\mu_5(\lambda N_1-\lambda_0P-\lambda_1PE)+\mu_6(\psi P-\psi_0E)$$

where $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$ and μ_6 are adjoin variables for B_1, B_2, N_1, N_2, P, E. The differential equation is as follows

$$\frac{d\mu_1}{dt}=-\left(s\left(1-\frac{B_1}{L_1}\right)-\alpha_1N_1-\lambda_2B_1P+B_1\left(-\frac{s}{L1}-\lambda_2P\right)-q_1M_1\right)\mu_1$$
$$-N_1\pi_1\alpha_1\mu_3-e^{-\delta t}(-M_1^2q_1^2v_1B_1+(-B_1M_1q_1v_1+p_1)q_1M_1)$$
$$\frac{d\mu_2}{dt}=-\left(s\left(1-\frac{B_2}{L_2}\right)-\alpha_2N_2-\frac{B_2s}{L_2}-q_2M_2\right)\mu_2-N_2\pi_2\alpha_2\mu_4$$
$$-e^{-\delta t}(-M_2^2q_2^2v_2B_2+(-B_2M_2q_2v_2+p_2)q_2M_2)$$
$$\frac{d\mu_3}{dt}=B_1\alpha_1\mu_1-\left(r\left(1-\frac{N_1}{K_1}\right)+\pi_1\alpha_1B_1+\gamma N_2\right)\mu_3+\frac{N_1r\mu_3}{K_1}+N_2\gamma\mu_4 \tag{8}$$
$$-\lambda\mu_5$$
$$\frac{d\mu_4}{dt}=B_2\alpha_2\mu_2-N_1\gamma\mu_3-\left(r\left(1-\frac{N_2}{K_2}\right)+\pi_2\alpha_2B_2-\gamma N_1\right)\mu_4+\frac{N_2r\mu_4}{K_2}$$
$$\frac{d\mu_5}{dt}=B_1^2\lambda_2\mu_1-\psi\mu_6-(-E\lambda_1-\lambda_0)\mu_5$$
$$\frac{d\mu_6}{dt}=\psi_0\mu_6+P\lambda_1\mu_5$$

with transversal conditions

$$\mu_i(t_f)=0, where\ i=1,2,\dots,6 \tag{9}$$

Optimum conditions

$$\frac{\partial H}{\partial M_1}=-q_1B_1\mu_1+e^{-\delta t}(-B_1^2q_1^2v_1M_1+(-B_1M_1q_1v_1+p_1)q_1B_1-c_1)=0$$
$$\frac{\partial H}{\partial M_2}=-q_2B_2\mu_2+e^{-\delta t}(-B_2^2q_2^2v_2M_2+(-B_2M_2q_2v_2+p_2)q_2B_2-c_2)=0 \tag{10}$$

Using the conditions of the control variable (5) and the equation (9), so that the optimal control variable is as follows

$$M_1^*=\min\left\{1, max\left\{0,\frac{1}{2}\frac{B_1e^{-\delta t}p_1q_1-q_1B_1\mu_1-e^{-\delta t}c_1}{e^{-\delta t}B_1^2q_1^2v_1}\right\}\right\}$$
$$M_2^*=\min\left\{1, max\left\{0,\frac{1}{2}\frac{\left(B_2e^{-\delta t}p_2q_2-q_2B_2\mu_2-e^{-\delta t}c_2\right)}{e^{-\delta t}B_2^2q_2^2\,v_2}\right\}\right\} \tag{11}$$

3.3 Numerical Solution

The numeric simulation in this article uses the parameter values and the initial compartment values that are presented in Table 3 and table 4.

Table 3: Parameter Values

Parameter	Value	Parameter	Value
s	0.6	L_1	50
L_2	30	α_1	0.0001

α_2	0.0002	λ	0.2
λ_0	0.1	λ_2	0.0002
r	0.1	K_1	100
K_2	60	π_1	0.004
π_2	0.001	γ	0.0005
λ_1	0.01	ψ	0.1
ψ_0	0.2	tf	40
δ	0.01	c_1	0.05
c_2	0.07	p_1	1
p_2	1.5	v_1	1.75
v_2	1.45	q_1	0.8
q_2	0.2		

Table 4: Initial Compartment Values

Compartment	$B_1(0)$	$B_2(0)$	$N_1(0)$	$N_2(0)$	$P(0)$	$E(0)$
Value	35	25	100	25	40	20

3.3.1 Numerical Simulation for Forestry dynamic Model

In this article, the forestry dynamic model of the system (1) represents the condition that the absence of logging and tourism that make a profit. Therefore, the value of the rate of reduction $q_1$and q_2 equal to zero.

From the parameter value in the table (3.2), the condition for which there is a solution that the all variables are not zero value $F^*(B_1^*, B_2^*, N_1^*, N_2^*, P^*, E^*)$ is met and the value of F^* :

$$B_1^* = 24.94756656, B_2^* = 29.73912543, N_1^* = 113.0537073$$
$$N_2^* = 26.08745651, P^* = 57.98638313, E^* = 28.99319157$$

Using these parameters, The point F^* Meet Global stability requirements on theorem 1 thus F^* have global stability. The Eigenvalues in the Matrix Jacobian at F^* are $\Lambda_1 = -0.58870, \Lambda_2 = -0.29496 + 0.22129, \Lambda_3 = -0.29496 - 0.22129, \Lambda_4 = -0.10001, \Lambda_5 = -0.05652, \Lambda_6 = -0.59478$. All of these eigen values have a negative value so that at F^*has a local steady point.

By using parameter values in the table 3 and the initial values in table 4 can be formed graphs on population dynamics over time.

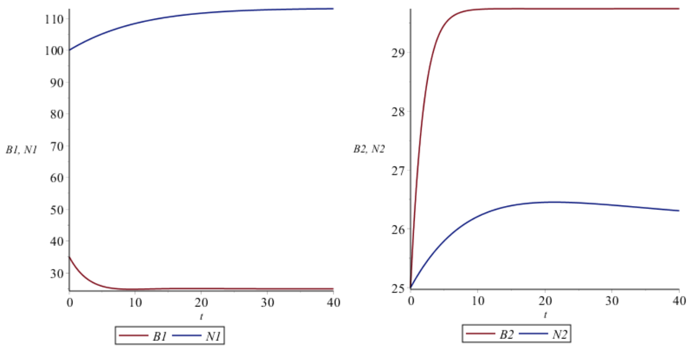

Figure 2: Population and their forest dynamics

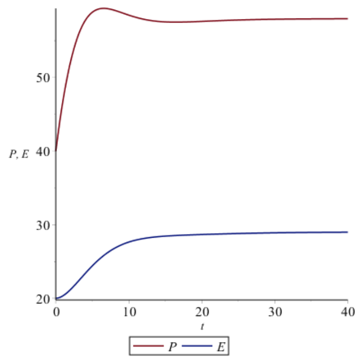

Figure 3: Population pressure and economic incentives dynamics

In the figure 2 shows the population dynamics between forest biomass and the population that manages the forest. On forest biomass managed by indigenous peoples B_1 have decreased to the value B_1^*, that has a difference of value far with its carrying capacity L_1. It is different from B_2. Value B_2 increase and toward to value B_2^* That has a sufficient difference in value close to its carrying capacity, namely L_2. This is due to the population pressure P which reduces carrying capacity N_1.

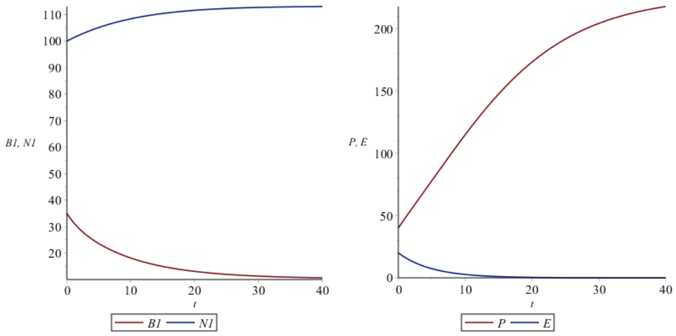

Figure 4: Dynamical population of B_1 and P with value $\psi = 0$

If there is no growth in economic incentives or value $\psi = 0$ then the value of P will be greater and further reduce the carrying capacity of B_1Thus lowering the amount B_1 as in the figure 4.

3.3.2 Numerical Simulation for Forestry dynamic Model with forestry and tourism factor

Logging and tourism factors were added to the model (1) to model (3). The control variables $M_1$and M_2 those that have been specified in the equation (11) are performed a numerical simulation to indicate the value of M_1 and M_2 for each time interval with the parameter values in the table 3.

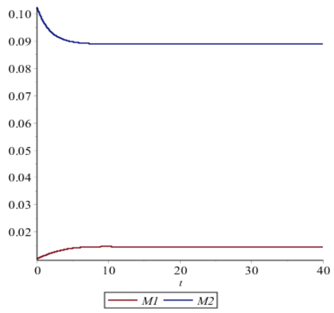

Figure 5: Dynamic Variable Control M_1 and M_2

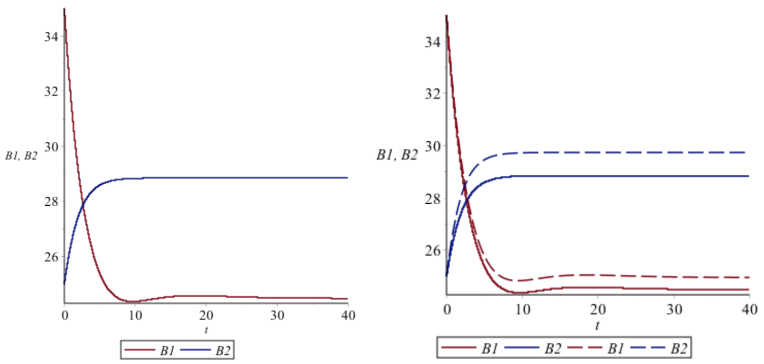

Figure 6: Forest Biomass dynamics after the logging and tourism factors were given

The figure 5 shows the value of M_1 and M_2 at any time lapse. Value of M_1 is convergent on 0.09 while value of M_2 is convergent on 0.012. The figure 6 shows the differences in forest biomass dynamics before and after the logging and tourism factors were given.

4. Conclusion

Based on the results of the analysis, the forestry model with the distribution of forest areas, indigenous peoples, non-indigenous peoples, population pressures and economic incentives provide a single stable solution that is when all the fixed point variables on the forestry model have a non-zero value and that point has both local and global stability if it meets certain conditions. The optimal control model of the forestry model by forest utilization in the form of deforestation in non-indigenous forests and tourism in indigenous peoples has optimal solutions. Numerical simulation results on the forestry model with optimal control indicate that with the use of forests with logging and tourism, lowering the number of forest biomass. However, the forest is still sustainable. Forest utilization according to control will maximize the profit of logging and tourism in the related forest.

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