



Implementation of Open Jackson Queueing Network on Customer Service and Ticket Service at Pasar Senen Train Station Jakarta

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Abstract

The increase in the number of users of train services causes a buildup in various service facilities, which affects the quality and efficiency of services. This study aims to optimize the service system at Pasar Senen Station by applying the open Jackson queueing network model using the First Come First Serve discipline. The data used includes external arrival times, service times, and cost components at three main facilities: customer service locket, ticket counter locket, and ticket vending machine locket. The results showed that optimal performance was achieved at the ticket counter (M/M/3) and ticket vending machine (M/M/2) with a busy level of 82.14% and 62.66% respectively. Simulation of reducing the number of servers in customer service to two (M/M/2) results in a busy level of 77.95% and lower total costs.

Keywords: service system, train station, open Jackson queueing network, cost.

1. Introduction

Public service systems, especially in the transportation sector, play an important role in supporting community mobility. Service efficiency not only increases user convenience, but also strengthens trust in public services. One of the challenges that often arises is the accumulation of customers which results in long queues and long waiting times in a service. According to Ratminto (2015), service involves interaction between customers and the facilities provided, which provide customer needs. The phenomenon of long queues often arises when the volume of customers exceeds the available service capacity.

Pasar Senen Railway Station, as one of the stations in Jakarta, provides customer service, ticket counter, and ticket vending machine to fulfill various customer needs, including ticket cancellation, direct ticket purchase, and schedule change. With the increasing volume of customers post-Covid-19 pandemic, there has been a buildup at the service facilities. According to the Central Bureau of Statistics (BPS), the number of railway customers in 2023 reached 371 million, showing a significant increase over the previous period. This backlog not only affects operational efficiency, but also increases operational costs. Over-provision of service capacity can result in high costs. Conversely, if service capacity is inadequate, it can lead to long waiting times and other negative consequences. By conducting an analysis, it is possible to find a balance between service costs and waiting times (Hillier and Lieberman, 2015).

Stacking in a service system often involves more than one interconnected facility. In these situations, simulation can be used to predict real conditions and obtain analysis results. The open Jackson queueing network model can analyze the flow of customers moving between service facilities with a certain probability (Sigman, 1990). Previous studies have applied this model in various contexts, which can be used as a reference for this study. Putri and Enfrayanto (2019) *et al.* (2020) used the open Jackson queueing network to analyze waiting times at health facilities, while Alam *et al.* (2021) utilized the open Jackson queueing network model in optimizing resource allocation in traffic vehicle networks. Javid *et al.* (2022) addressed a multi-service system design problem with facilities modeled as an open Jackson network.

Several studies related to costs in service systems show the importance of analyzing efficiency and the relationship between cost components. Muhajir and Binatari (2017) found that in an insurance company, adding one server to the customer service facility can reduce service costs to reach optimal conditions. Meanwhile, Vijay Prasad *et al.* (2020)

analyzed the sensitivity between service costs, waiting costs, and total expected costs in a multi-server model, thus providing insight into achieving an efficient cost balance.

Based on this description, this study aims to analyze the service system at Pasar Senen Train Station, Jakarta. This research involves several facilities, such as customer service, ticket counter, and ticket vending machine, by applying the open Jackson queueing network model to analyze the flow of customer movement between these facilities. In addition, this research also considers the operational costs incurred from each service facility, in order to evaluate cost efficiency in supporting overall system performance by determining the optimal number of servers at each facility.

2. Service System

2.1. Steady State

A steady state condition in a queueing system is reached when the arrival rate (λ) is smaller than the service speed (μ), with the service facility utility rate (ρ) which must be smaller than 1. To calculate the steady state value, the arrival rate is calculated based on the number of customers arriving in a time interval, the service rate (μ) is calculated based on the number of customers served in the total service time, and the number of servers used to serve customers (c) (Taha, 2017).

$$\rho = \frac{\lambda}{c\mu}$$

2.2. Service Model $[M/M/c]:[FCFS/\infty/\infty]$

The M/M/c service model is a queueing system with Poisson-distributed arrivals and exponentially distributed services, using c servers ($c > 1$) (Gross *et al.*, 2018). The system operates under the FCFS (First-Come-First-Served) queueing discipline, system has unlimited capacity and unlimited input source size. Below are the performance measures of the service system that are calculated.

1. Probability of no customer in the system

$$P_0 = \frac{1}{\sum_{n=0}^{c-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^c}{c!} \frac{1}{1 - \frac{\lambda}{c\mu}}}$$

2. The average number of customers in the system

$$L_s = L_q + \rho$$

3. The average number of customers in the queue

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^c \rho}{c! (1 - \rho)^2} P_0$$

4. The average waiting time in the system.

$$W_s = W_q + \frac{1}{\mu}$$

5. The average waiting time in the queue.

$$W_q = \frac{L_q}{\lambda}$$

2.3. Kolmogorov Smirnov Test

The Kolmogorov-Smirnov (KS) test is used to determine if the sample data follows a specific theoretical distribution. Steps in the Kolmogorov-Smirnov Test:

1. Determining Hypotheses:

H_0 : The data follows a specific distribution.

H_1 : The data does not follow a specific distribution.

2. Significance Level:

$$\alpha = 0.05$$

3. Test Statistic:

The KS test statistic is the largest absolute difference between the sample cumulative distribution function $F_s(x)$ and the theoretical cumulative distribution function $F_t(x)$, denoted as D (maximum deviation).

$$D = \max |F_s(x_i) - F_t(x_i)|$$

4. Decision:

a. H_0 is accepted at a significant level of $\alpha = 0,05$, if the value of $D \leq D^*(\alpha)$ or $Asym.Sig > \alpha$, which means that the distribution of the observed sample data has the same distribution as the theoretical distribution.

b. H_0 is rejected at a significant level of $\alpha = 0,05$, if the value of $D > D^*(\alpha)$ or $Asym.Sig < \alpha$, which means that the distribution of the observed sample data does not match the theoretical distribution.

2.4. Open Jackson Queueing Network

The open Jackson network model allows customers to enter the system through service facilities with external arrival rates (γ_i) which follows a Poisson distribution. After being served, customers can move to another facility with a probability of P_{ij} or exit the system with probability $P_{i, keluar}$. The total displacement probability is calculated as $\sum_{j=1}^N P_{ij} + P_{i, out} = 1$ (Harchol Balter, 2013). This displacement probability matrix is called the Jackson transition matrix.

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1N} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2N} \\ P_{31} & P_{32} & P_{33} & \cdots & P_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{N1} & P_{N2} & P_{N3} & \cdots & P_{NN} \end{bmatrix}$$

Servers in the facility work independently with an exponential service distribution, following the FCFS (First Come First Served) queuing discipline. The system has no capacity limit, so no facility is blocked. The total arrival rate at each facility (λ_j), consists of external arrivals (γ_j), and internal arrivals (b_j). Internal arrivals are calculated as $b_j = \sum_{i=1}^N \lambda_i P_{ij}$, $1 \leq j \leq N$, So that The total arrival rate (Kulkarni, 2011).

$$\lambda_j = \gamma_j + \sum_{i=1}^N \lambda_i P_{ij}, 1 \leq j \leq N$$

In matrix form, the total arrival rate is:

$$\lambda = \gamma(I - P^T)^{-1}$$

with I as the identity matrix.

System stability is tested by calculating utilization (ρ), which must satisfy $0 < \rho < 1$. to achieve a steady-state condition, the next step is to analyze the performance of the service system. The average number of network system customers (L) as follows

$$L = \sum_{i=1}^N L_{si}$$

and the average length of customers in the network system (W), calculated as follows.

$$W = \frac{L}{\sum_{i=1}^N \gamma_i}$$

2.5. Service System Cost Model

The costs incurred in a service involve the cost of offering services and the cost of delays in offering services (customer waiting time) (Heizer & Render, 2011).

a. Cost Waiting Time

Waiting time costs arise as a result of the time spent by customers while getting service, the cost of waiting in units of time (EWC) is calculated as follows.

$$EWC = C_w L_s$$

where, C_w he cost of waiting borne by customers.

b. Service Cost

Cost of service where the costs incurred to provide services to customers (ESC), are calculated as follows.

$$ESC = C_s c$$

where, service operation costs (C_s).

Based on the calculation of service operation costs and waiting costs at each facility, the optimal number of servers for each facility can be determined. By reviewing the optimal number of servers is the one that provides the minimum total service system cost value. The total cost of the service system is calculated as follows.

Based on the calculation of service operation costs and waiting costs at each facility, the optimal number of servers for each facility can be determined. By reviewing the optimal number of servers is the one that provides the minimum total service system cost value. The total cost of the service system (ETC) is calculated as follows.

$$ETC = ESC + EWC$$

3. Result and Discussion

3.1. Service Network System at Pasar Senen Station

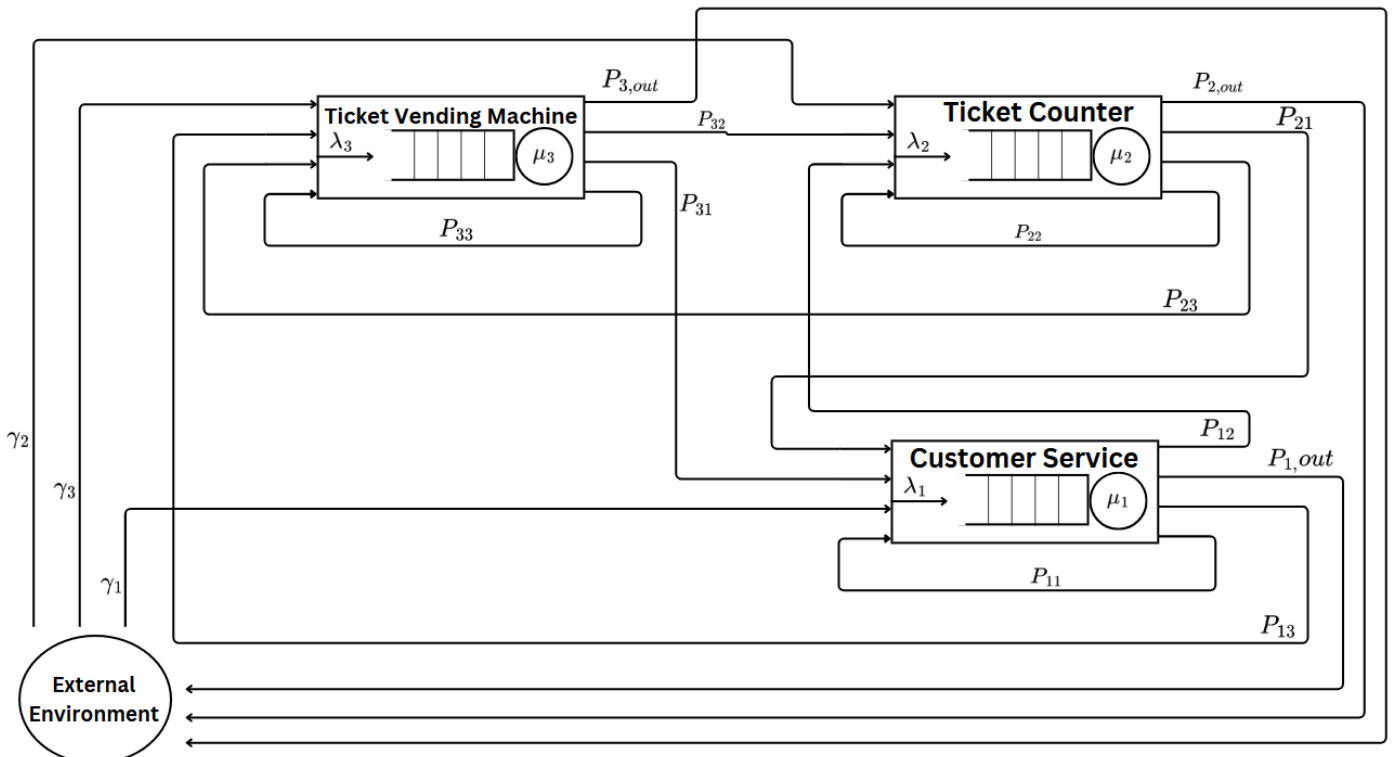


Figure 1: Schematic of the open Jackson queueing network at the customer service facility, ticket counter, and ticket vending machine at Pasar Senen Station.

The use of the open Jackson queueing network model in this study begins by calculating the probability of customer movement between service facilities at Pasar Senen Station. The probability of movement is calculated based on the number of customers who move from facility i to j , compared to the total customers served at the facility. Switching probability This is organized in the form of the following transition matrix (P):

$$P = \begin{bmatrix} 0,004525 & 0,099548 & 0,013575 \\ 0,035025 & 0,010007 & 0,022159 \\ 0,060241 & 0,228916 & 0,006024 \end{bmatrix}$$

The probability value is needed to calculate the total arrival rate, with the matrix calculation yielding the following result.

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0,330478 \\ 0,895690 \\ 0,302363 \end{bmatrix}$$

3.2. Steady State Size

To maintain the stability of the service system and ensure that there is no significant change in the average arrival or average service, it is necessary to test the steady state condition (ρ). This steady state is obtained from the calculation of the total arrival speed and service speed. The steady state value is shown in Table 1.

Table 1: Steady state size

Facilities	λ	μ	c	ρ
Customer Service	0.330478	0.21199	3	0.519644
Ticket Counter	0.895690	0.363471	3	0.821423
Ticket Vending Machine	0.302363	0.241279	2	0.626583

ρ value below 1 indicates that the system satisfies the steady-state condition.

3.3. Distribution Fit Test

Before processing the data, preliminary testing is conducted to determine the appropriate service system model for each facility. The arrival time and service time are tested using the One-Sample Kolmogorov-Smirnov test with the assistance of IBM SPSS 26 software. The arrival time data is assumed to follow a Poisson distribution, while the service time data is assumed to follow an Exponential distribution.

Table 2: Arrival test using Poisson distribution

Facilities	D	$D^*(\alpha)$	Asym. sig	α	Poisson
Customer Service	0.300	0.48343	0.556	0.05	Yes
Ticket Counter	0.193	0.48343	0.956	0.05	Yes
Ticket Vending Machine	0.148	0.48343	0.998	0.05	Yes

Table 3: Service test using Exponential distribution

Facilities	D	$D^*(\alpha)$	Asym. sig	α	Exponential
Customer Service	0.419	0.48343	0.172	0.05	Yes
Ticket Counter	0.228	0.48343	0.860	0.05	Yes
Ticket Vending Machine	0.293	0.48343	0.583	0.05	Yes

3.4. Service Model and Service System Performance Measure Analysis

The three customer facilities arrive with a Poisson-distributed arrival pattern and are served with an Exponential distribution pattern according to the First Come First Serve principle. The service model at the customer service is $[M/M/3]: [FCFS/\infty/\infty]$, ticket counter is $[M/M/3]: [FCFS/\infty/\infty]$, the ticket counter is $[M/M/3]$, and ticket vending machine is $[M/M/2]: [FCFS/\infty/\infty]$.

The results of the system performance analysis at each service facility are shown in Table 4.

Table 4: Results of the analysis of service system performance measures at each facility.

Facilities	P_0	L_s	L_q	W_s	W_q
Customer Service	0.594196	$1.559272 \approx 2$	$0.000340 \approx 1$	4.718233	0.001029
Ticket Counter	0.437446	$2.469785 \approx 3$	$0.005517 \approx 1$	2.757411	0.006160
Ticket Vending Machine	0.522890	$1.257200 \approx 2$	$0.004033 \approx 1$	4.157917	0.013338

The total number of customers in the network system is about $5.286257 \approx 6$ customers/minute. In addition, the average time spent by customers in the network system is about 4.480683 minutes/customer.

3.5. Service System Cost Analysis

- The cost of waiting borne by customers is assumed to be based on the average income per capita of DKI Jakarta according to BPS with the assumption of effective working hours for 8 hours. Average The average annual per capita income of DKI Jakarta is IDR 316,536,428, then the cost of waiting per minute (C_w) is IDR 1,831.808032 /minute.
- Service operation costs consist of three categories: direct labor costs, direct material costs (such as work tool facilities), and overhead costs (electricity costs). The officer's salary is IDR 6.000.000 / month assuming working hours for 7 hours for customer service and 6 hours for ticket counter. The cost of work tools is assumed to follow the average price in the online shop, and the electricity costs incurred are assumed to be at an electricity tariff of IDR 1,444.70 per kWh (PLN electricity tariff class B-2/TR). The following service costs for each facility are shown in Table 5.

Table 5: Results of the analysis of service system performance measures at each facility.

Facilities	Labor cost (IDR)	Facility cost (IDR)	Electricity Cost (IDR)	Total (IDR)
Customer Service	649.350649	2.533295	635.688	1.287.560944
Ticket Counter	757.575757	3.757610	674.674900	1.436.008267
Ticket Vending Machine	-	1.826484	650.115000	651.941484

Based on the calculation of service operation costs and waiting costs at each facility, the optimal number of servers for each facility can be determined. By reviewing the optimal number of servers is the one that provides the minimum total service system cost

Table 6: Total cost of the service system based on each number of customer service servers

c	C_s	C_w	L_s	ESC	EWC	ETC
2	1,235.268771	1,831.808032	1.563950	2,575.121888	2,864.856172	5,439.978060
3	1,235.268771	1,831.808032	1.559272	3,862.682832	2,856.286974	6,718.969806
4	1,235.268771	1,831.808032	1.558943	5,150.243776	2,855.684309	8,005.928085
5	1,235.268771	1,831.808032	1.558932	6,437.804720	2,855.664159	9,293.468879
6	1,235.268771	1,831.808032	1.558932	7,725.365664	2,855.664159	10,581.029823

Table 7: Total service system cost based on each number of ticket counter servers

c	C_s	C_w	L_s	ESC	EWC	ETC
2	1,235.268771	1,831.808032	1.563950	2,575.121888	2,864.856172	5,439.978060
3	1,235.268771	1,831.808032	1.559272	3,862.682832	2,856.286974	6,718.969806
4	1,235.268771	1,831.808032	1.558943	5,150.243776	2,855.684309	8,005.928085
5	1,235.268771	1,831.808032	1.558932	6,437.804720	2,855.664159	9,293.468879
6	1,235.268771	1,831.808032	1.558932	7,725.365664	2,855.664159	10,581.029823

Table 8: Total service system cost based on each number of ticket vending machine servers

c	C_s	C_w	L_s	ESC	EWC	ETC
2	1,235.268771	1,831.808032	1.563950	2,575.121888	2,864.856172	5,439.978060
3	1,235.268771	1,831.808032	1.559272	3,862.682832	2,856.286974	6,718.969806
4	1,235.268771	1,831.808032	1.558943	5,150.243776	2,855.684309	8,005.928085
5	1,235.268771	1,831.808032	1.558932	6,437.804720	2,855.664159	9,293.468879
6	1,235.268771	1,831.808032	1.558932	7,725.365664	2,855.664159	10,581.029823

Based on the calculation of the total cost per minute at customer service in Table 5, the optimal number of servers is 2 servers, which is IDR 5,439.98, with the minimum cost compared to the 3 servers currently used. Meanwhile, for ticket counters and ticket vending machines, with 3 servers each, they are already in optimal conditions.

4. Conclusion

Based on the results of research applied to customer service facilities, ticket counters, and ticket vending machines at Pasar Senen Railway Station, the application of the open Jackson queueing network model produces a service system model that can describe the workflow and performance of each facility. The analysis results show that the average number of customers in the system is about 6 customers per minute, with the average time spent by customers in the system is 4.48 minutes.

The service model applied to the customer service facility is $[M/M/2]: [FCFS/\infty/\infty]$ with a busy level of 77.95% and a service fee of IDR 5,439.98 per minute. For the ticket counter, the model applied is $[M/M/3]: [FCFS/\infty/\infty]$ with a busy level of 82.14% and a service cost of IDR 8,832.20 per minute. As for the ticket vending machine, the model applied is $[M/M/2]: [FCFS/\infty/\infty]$ with a busy level of 62.66% and a service cost of IDR 3,606.83 per minute.

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