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Application of Queue Theory in Campus Transportation at Padjadjaran Jatinangor University Using a Multiserver Queue System Model

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Abstract

Trasnportation system on campus is an important aspect that supports the mobility of the Academic Community and the relations or partners of Padjadjaran University. Currently, Padjadjaran Jatinangor University provides several public transportation facilities that can be used around campus area, namely conventional motorcycle taxis, Beam electric bicycles, and campus transportation in the form of buses. Based on the results of a survey conducted by the author, campus transportation is a facility of public transportation that is more often used and in demand by the Academic Community compared to the other two facilities of public transportation. This study aims to analyze the performance of the passenger queuing system on that campus transportation using a multiserver queue system model. Data in the arrival rate of bus passenger (λ) and the rate of bus service (μ) were collected through direct observation. The results of the study showed that during the operating hours at 07:00-08:00, routes A, B, and C are optimal with number of buses as many as 5, 5, and 6 respectively. Then, during operating hours at 09:45-10:45, routes B and C are optimal with number of buses each as many as 3 buses. As for route A, it is necessary to reduce the number of buses by 1 piece. Then during operating hours at 13:00-14:00, all routes need to be reduced to 1 bus each.

Keywords: Transportation, Queue theory, Multiserver, Resource allocation

1. Introduction

Queue is a social phenomenon that often occurs in everyday life. This phenomenon occurs due to service facilities that cannot compensate for customer arrival rates. In other words, the average customer arrival rate is faster than the average service rate. According to Aarthi and Sumitha (2022), factors such as limited service capacity, customer arrival rate, and service time can lead to queue.

Taha (2017) mentioned that the queue phenomenon does not only occur in humans. The process of unloading the ship of goods in the port, the stopping cars at traffic signs, the goods waiting for the packaging process on its application involves queue theory. This understanding provides insight that the application of queue theory is very broad. Theory of queue itself was first applied to telephone and communication queue system by Carl Gustav Jacob Erlang, a Danish engineer and mathematician.

Research that discusses the application of multiserver queue system models has been done a lot. Chan et al. (2023) conducted research on the implementation of the multiserver *M/M/s* single-phase queue system model in a supermarket. The result showed that the addition of two cashiers during rush hour will make the service optimal. Ferianto et al., (2016) optimized the service in gas station using the *M/M/s* multiserver queue system model involving operational cost factors, the result showed that service with three servers will balance operational costs and also the quality of the performance queue. Then Dehi et al. (2023) conducted research on the service of motorcycle services that followed the distribution of General. The results can be concluded that the queue system found in the optimal motor service uses 5 servers. Then Suban et al. (2018) analyzed the service on the entrance cross of two-wheeled vehicles in the market Alok Maumere. The results of the study found effective service time, which is 15 seconds.

In this study, queue theory applied to transportation system of Padjadjaran Jatinangor University Campus transportation system. The objective is to find optimal number of buses so that the passenger queue system does not experience significant buildup. The model used is a multiserver queue system model, where the service is performed by several buses.

2. Literature Review

2.1 Basic Components of Queue System

According to Hillier & Lieberman (2001), here are some basic components of the queue system:

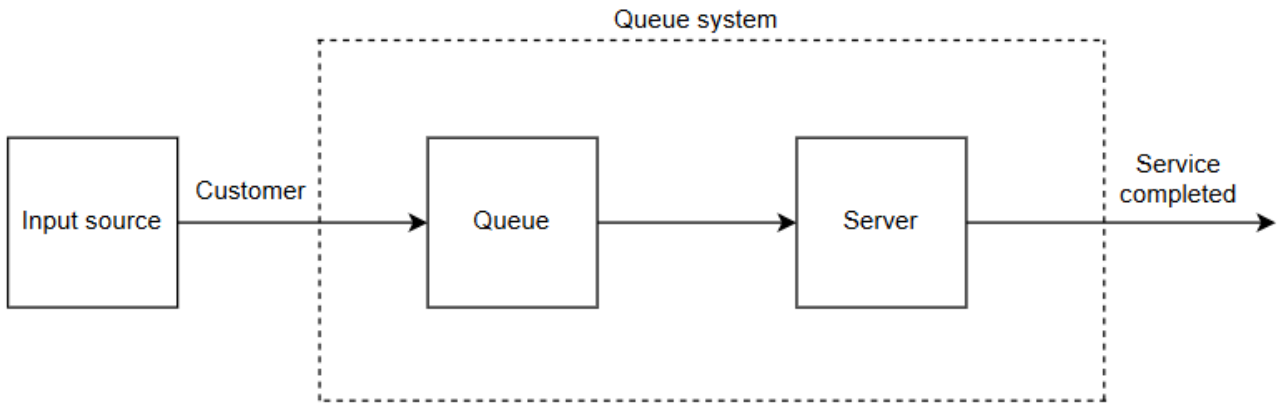

Figure 1: Basic components of queue system

The first component is input source, it is the source of the customer population that has the potential to fit into queue system. One characteristic that can be observed in this component is the size of its population. The size of the customer population can be limited or unlimited. The size of the customer population is limited means that maximum number of customers who can potentially enter into the queue system can be calculated. While the reduction of the unlimited customer population means that maximum number of customers who have the potential to enter into a relative queue system cannot be calculated because the number of its population is too large.

The second component is queue, the process by which the customer must wait some time before being served. The characteristic that can be observed in this component is that the queue capacity can be limited or unlimited depending on the queue system itself. Generally, the majority of queue system models assume unlimited queue capacity. However, in situations where the queue capacity is extremely limited, it is advisable to apply the assumption of limited queue capacity.

The last component is a server, it is someone or something that serves the customer within a certain period of time. The observable characteristic of this component is the service discipline, it is the rules that govern how customers are served. According to Heizer & Render (2011), there are several service disciplines: First Come First Served (FCFS), Last Come First Served (LCFS), Priority Service (PS), and Service In Random Order (SIRO).

2.2 Model Queue System $(G/G/s)$: $(FCFS/\infty/\infty)$

Queue theory can provide significant advantages in a variety of applications. In his research, Adolphs et al. (2024) said that the effectiveness of a queue system model relies on accurate assumptions of system behavior. We should start by accurately assuming the arrival pattern, service time pattern, service discipline, input source, and queue capacity. Each model of the queue system has different assumptions depending on the behavior of the queue system. In this study, the model used in this research is $(G/G/s)$: $(FCFS/\infty/\infty)$.

This model of queue system assumes customer arrival patterns and system service follow a General distribution. The number of servers is s and the discipline of the service is First Come First Served (FCFS). This means that first-time customers will get service first. The source of input and the capacity of the queue have an unlimited amount. According to Atkinson & Walrand (1990), the calculation of queue performance in this model can use the following formula:

Empty system probability (P_0):

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!} \frac{1}{1 - \frac{\lambda}{s\mu}} \right]^{-1} \tag{1}$$

Where λ is the average passenger arrival speed, μ is the average service duration, and s is the number of buses. Next, to find for the average number of customers in the queue (L_q) can use the following formula:

$$L_q = \frac{P_0(\lambda/\mu)^s \rho}{s!\,(1-\rho)^2} \frac{\mu^2 Var[p] + \lambda^2 Var[k]}{2} \tag{2}$$

Where $\rho = \lambda/\mu$, $var[p]$ is a variance of service speed, and $var[k]$ is a variance of passenger arrival speed. Then, to search for the average number of customers in the system (L), the average waiting time for passengers in the queue (W_q), and the average waiting time for passengers in the system (W) can be obtained from the following Little's Formula:

$$L = L_q + \frac{\lambda}{\mu} \tag{3}$$

$$W_q = \frac{L_q}{\lambda} \tag{4}$$

$$W = W_q + \frac{1}{\mu} \tag{5}$$

2.3 Goodness-of-Fit Test

Goodness of fit test is performed to ensure that the selected distribution is in harmony with the underlying data. There are several commonly used testing methods. Among them are the Kolmogorov-Smirnov test, the chi-square test, the Anderson-Darling test, the Cramer-von Mises test, etc. In this study, the distribution test used is the chi-square test.

The first step in this test is to calculate the observed frequency (O_i), which is to calculate the number of appearances of each category in the observation data. The second step is to calculate the expectation value or frequency expectation ($\hat{f}_i$), i.e. the expected frequency of each category. The third step is to calculate the value of Chi-Square (χ^2) through the following formula:

$$\chi^2 = \sum \frac{\left(O_i - \hat{f}_i\right)^2}{\hat{f}_i} \tag{6}$$

The final step is to compare the calculated value of the Chi-Square Table with the value of the chi-square table. If the calculated chi-square value is greater than the value on the chi-square table, then the zero hypothesis is rejected. This means that the sample distribution does not match the expected theoretical distribution: it can be assumed to follow General distribution.

3. Materials and Methods

3.1. Materials

The study used primary data, namely the rate of passengers arriving in the form of how many bus passengers arrive at the station at the stop during a given unit of time. Then the service rate is how many passengers are served. Not only that, the study also uses secondary data obtained from the manager in the form of the number of buses operating during busy hours.

3.2. Methods

The purchase aims to find the optimal number of buses so that the passenger queue system of Padjadjaran Jatinangor University campus does not experience significant buildup. The following steps are:

1) Data collection of passenger arrival rate and bus service rate during five days of lecture,
2) Perform the goodness of fit test of passenger arrival pattern and bus service duration pattern using formula 6,
3) Determine the appropriate queue system model based on the pattern of passenger arrival distribution and the pattern of distribution of bus service duration,
4) Calculate the performance of the bus passenger queue system through formulas 1-5

5) Evaluate the results of the performance calculation of the bus passenger queue system that has been done in step 4,
6) If the performance of the queue system is not optimal, then the simulation of the calculation is done by adding or reducing the number of buses and back to step 5,
7) If the performance of the queue system is optimal, then the conclusion is taken.

4. Results and Discussion

The research data used are as follows:

Table 1: Average and variance of arrival and bus passenger service

No	Bus routes	Operating hours	Average rate of passenger arrival (person/minute)	Variance of passenger arrival	Average service rate (person/minute)	Variance of service
1.	A	07:00-08:00	2.85	2.54	1.02	0.16
		09:45-10:45	1.22	1.13	0.76	0.09
		13:00-14:00	0.79	0.52	0.63	0.11
2.	B	07:00-08:00	2.68	2.54	0.92	0.12
		09:45-10:45	1.78	1.13	0.96	0.11
		13:00-14:00	0.88	0.52	0.69	0.18
3.	C	07:00-08:00	3.53	2.54	1.02	0.10
		09:45-10:45	1.51	1.13	0.73	0.13
		13:00-14:00	0.78	0.52	0.52	0.05

4.1 Goodness-of-Fit Test Distribution of Passenger Arrival Rate

Passenger arrival data is made into several categories of arrival range classes. From each category of arrival range is sought the frequency of its occurrence (O_i). Then, to find the average arrival of passengers, specify first the midpoint value of each category. The average arrival of passengers is obtained from the sum of the multiplication of the frequency of events and the value of the midpoint of each category, then the result is divided by the total frequency of events. So the results are obtained on teh following table 2-4:

Table 2: Calculation of goodness-of-fit test on the arrival of bus passengers at the operating hours at 7:00-8:00

Category of passenger arrival range	Occurred frequency (O_i)	Midpoint	Poisson probability	Expected frequency ($\hat{f}_i$)	$\frac{(O_i - \hat{f}_i)^2}{\hat{f}_i}$
40 – 79	11	60	0.09979628	2.9938884	21.40955654
80 – 99	9	90	0.695261665	20.85784995	6.741279942
100 – 199	4	110	0.202327817	6.069834518	0.705820714
120 – 159	6	140	0.002614237	0.078427114	447.1033499
Average arrival rate					91.66667

Table 3: Calculation of goodness-of-fit test on the arrival of bus passengers at the operating hours at 09:45-10:45

Category of passenger arrival range	Occurred frequency (O_i)	Midpoint	Poisson probability	Expected frequency ($\hat{f}_i$)	$\frac{(O_i - \hat{f}_i)^2}{\hat{f}_i}$
17 – 29	4	23	0.008341364	0.2502409	56.18862483
30 – 42	12	36	0.373116591	11.193498	0.058109264
43 – 55	9	49	0.56209834	16.86295	3.666380156
56 – 94	5	75	0.056442971	1.6932891	6.457454107
Average arrival rate					44.66667

Table 4: Calculation of goodness-of-fit test on the arrival of bus passengers at the operating hours at 13:00-14:00

Category of passenger arrival range	Occurred frequency (O_i)	Midpoint	Poisson probability	Expected frequency ($\hat{f}_i$)	$\frac{(O_i - \hat{f}_i)^2}{\hat{f}_i}$
4 – 12	5	8	0.002064712	0.0619414	393.6694
13 – 21	7	17	0.201316923	6.0395077	0.152752
22 – 30	17	26	0.622996849	18.689905	0.152798
31 - 57	8	44	0.173621461	5.2086438	1.495911
		Average arrival rate			25.75676

The chi-table value obtained for the passenger arrival distribution test is 5.99. Therefore, the test results show the H_0 expectable rejection because the chi-count value is greater than the chi-table. This means that the distribution of the arrival of passengers of the bus follows the distribution of General. Graphically, it can be seen as the following figures:

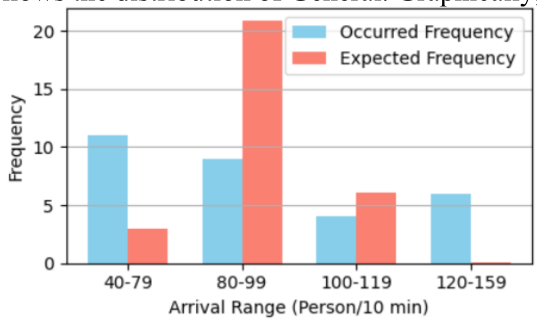

Figure 2: Distribution of bus passenger arrivals during operating hours 07:00 – 08:00

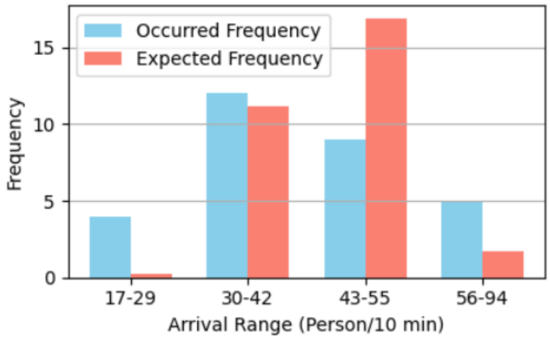

Figure 3: Distribution of bus passenger arrivals during operating hours 09:45 – 10:45

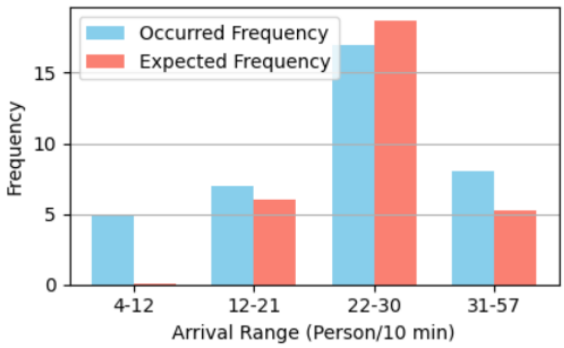

Figure 4: Distribution of bus passenger arrivals during operating hours 13:00 – 14:00

4.2 Goodness-of-Fit Test of Bus Service Duration

Just like passenger arrival pattern testing, the duration of bus service data is first created in several categories of duration range. Then, from each category of the duration range, find the frequency of events (O_i) and the value of the midpoint. The average duration of service is obtained from the sum of multiplication results between the frequency of events and the midpoint value of each category, then the result is divided by the total frequency of events. The results can be seen in the following table 5-7:

Table 5: Calculation of goodness-of-fit test duration of route bus service A

Category of service duration range	Occurred frequency (O_i)	Midpoint	Eksponential probability	Expected frequency ($\hat{f}_i$)	$\frac{(O_i - \hat{f}_i)^2}{\hat{f}_i}$
8 – 10	10	9	0.216168586	27.45341045	11.09594514
10 – 12	39	11	0.185378296	23.54304365	10.14811437
12 – 14	37	13	0.158973667	20.18965568	13.99665653
14 – 16	28	15	0.136330019	17.31391245	6.595416687
16 – 18	8	17	0.116911653	14.84777993	3.158188642
18 – 20	4	19	0.10025917	12.73291461	5.989500431
20 – 22	1	21	0.085978608	10.91928323	9.010864334
Average service duration					13.01
Exponent function parameters					0.08

Table 6: Calculation of goodness-of-fit test duration of route bus service B

Category of service duration range	Occurred frequency (O_i)	Midpoint	Eksponential probability	Expected frequency ($\hat{f}_i$)	$\frac{(O_i - \hat{f}_i)^2}{\hat{f}_i}$
8 – 10	12	9	0.201689179	29.849999	10.67411936
10 – 12	47	11	0.172845887	25.581191	17.93369832
12 – 14	50	13	0.148127434	21.92286	35.95907485
14 – 16	23	15	0.126943934	18.787702	0.944418464
16 – 18	9	17	0.108789856	16.100899	3.131673808
18 – 20	2	19	0.093231969	13.798331	10.08822147
20 – 22	4	21	0.079898993	11.825051	5.178110666
22 – 24	1	23	0.068472748	10.133967	8.232644725
Average service duration					12.96
Exponent function parameters					0.08

Table 7: Calculation of goodness-of-fit test duration of route bus service C

Category of service duration range	Occurred frequency (O_i)	Midpoint	Eksponential probability	Expected frequency ($\hat{f}_i$)	$\frac{(O_i - \hat{f}_i)^2}{\hat{f}_i}$
11 – 13	28	12	0.226386483	32.599654	0.648989
13 – 15	48	14	0.198249184	28.547883	13.25439
15 – 17	42	16	0.173609036	24.999701	11.56054
17 – 19	17	18	0.152031382	21.892519	1.093375
19 – 21	7	20	0.133135587	19.171525	7.727399
21 – 23	2	22	0.116588328	16.788719	13.02697
Average service duration					15.07
Exponent function parameters					0.07

The chi-table values obtained for consecutive A, B, and C bus service distribution tests are 11.07, 12.59, and 9.49. Therefore, the test results show the H_0 expectable rejection because the chi-count value is greater than the chi-table.

This means that the distribution of bus services follows the distribution of General. Graphically, it can be seen as the follows\ing figures:

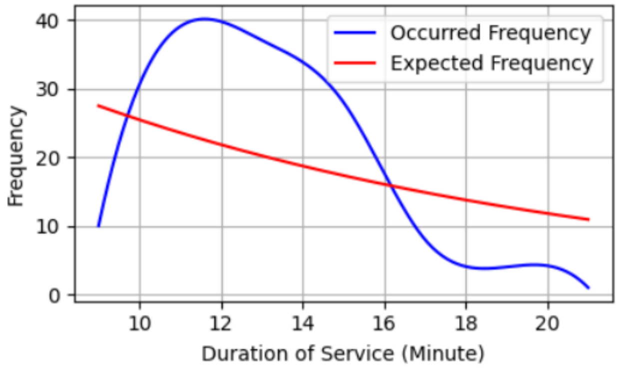

Figure 5: Distribution of bus service duration during operating hours 07:00 – 08:00

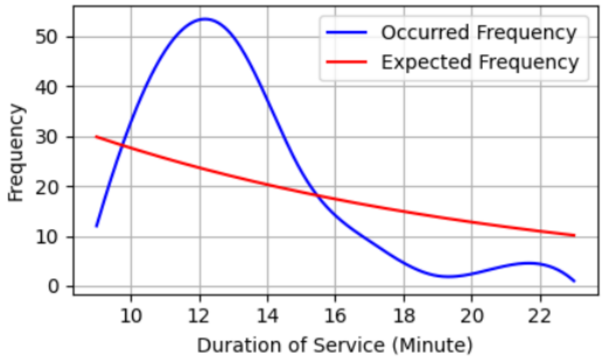

Figure 6: Distribution of bus service duration during operating hours 09:45 – 10:45

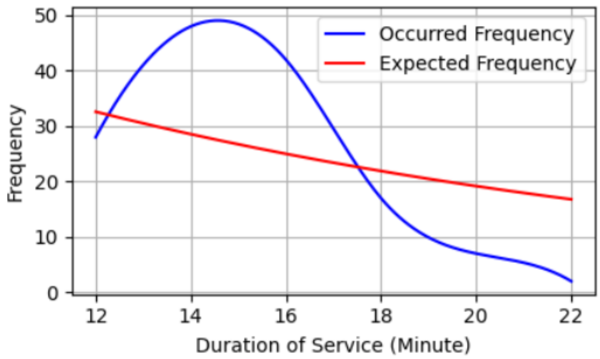

Figure 7: Distribution of bus service duration during operating hours 13:00 – 14:00

4.3 Results of the Performance of The Bus Passenger Queue System

Based on the testing of the passenger arrival distribution and bus service, the model of the queue system formed is $(G/G/s)$: $(FCFS/\infty/\infty)$. Next, the results of the performance calculation of passenger queues from each bus route can be seen in the following tables:

Table 8: The result of the performance calculation of the route A bus passenger queue system at operating 07:00-08:00 hours

s	$\rho = \lambda/\mu s$	P_0	L_q	L	W_q	W
3	0.9292	0.017	118.963	121.751	41.709	42.6864
4	0.6969	0.0511	10.1602	12.9479	3.5622	4.5396
5	0.5575	0.0589	2.454	5.2417	0.8604	1.8377
6	0.4646	0.0609	0.6707	3.4584	0.2351	1.2125
7	0.3982	0.0614	0.1827	2.9704	0.0641	1.0414

Table 9: The result of the performance calculation of the route A bus passenger queue system at operating 09:45-10:45 hours

s	$\rho = \lambda/\mu s$	P_0	L_q	L	W_q	W
2	0.8005	0.1108	2.4598	4.0609	2.0218	3.3377
3	0.5337	0.1869	0.2702	1.8713	0.2221	1.538
4	0.4003	0.1991	0.0522	1.6533	0.0429	1.3588
5	0.3202	0.2012	0.0105	1.6116	0.0087	1.3246

Table 10: The result of the performance calculation of the route A bus passenger queue system at operating 13:00-14:00 hours

s	$\rho = \lambda/\mu s$	P_0	L_q	L	W_q	W
2	0.6221	0.233	0.1447	1.3889	0.1837	1.763
3	0.4147	0.2804	0.0201	1.2643	0.0255	1.6049
4	0.3111	0.287	0.0035	1.2477	0.0044	1.5838
5	0.2488	0.288	0.0006	1.2448	0.0007	1.5801

Table 11: The result of the performance calculation of the route B bus passenger queue system at operating 07:00-08:00 hours

s	$\rho = \lambda/\mu s$	P_0	L_q	L	W_q	W
3	0.9659	0.0079	242.277	245.175	90.5521	91.6351
4	0.7244	0.0438	11.2367	14.1342	4.1997	5.2827
5	0.5795	0.0523	2.6675	5.5651	0.997	2.08
6	0.4829	0.0544	0.7391	3.6367	0.2763	1.3592
7	0.4139	0.055	0.2062	3.1038	0.0771	1.1601

Table 12: The result of the performance calculation of the route B bus passenger queue system at operating 09:45-10:45 hours

s	$\rho = \lambda/\mu s$	P_0	L_q	L	W_q	W
2	0.928	0.0373	21.0742	22.9302	11.8617	12.9063
3	0.6187	0.1356	1.1254	2.9814	0.6334	1.6781
4	0.464	0.1523	0.2226	2.0786	0.1253	1.1699
5	0.3712	0.1555	0.049	1.905	0.0276	1.0723

Table 13: The result of the performance calculation of the route B bus passenger queue system at operating 13:00-14:00 hours

s	$\rho = \lambda/\mu s$	P_0	L_q	L	W_q	W
2	0.6377	0.2212	0.213	1.4884	0.2427	1.6956
3	0.4251	0.271	0.0294	1.3047	0.0335	1.4864
4	0.3188	0.2781	0.0051	1.2805	0.0058	1.4588
5	0.2551	0.2791	0.0009	1.2762	0.001	1.4539

Table 14: The result of the performance calculation of the route C bus passenger queue system at operating 07:00-08:00 hours

s	$\rho = \lambda/\mu s$	P_0	L_q	L	W_q	W
4	0.8657	0.0161	73.7853	77.2481	20.8892	21.8695
5	0.6926	0.0271	13.1177	16.5804	3.7137	4.694
6	0.5771	0.0301	3.705	7.1678	1.0489	2.0292
7	0.4947	0.031	1.1317	4.5944	0.3204	1.3007
8	0.4328	0.0312	0.343	3.8057	0.0971	1.0774

Table 15: The result of the performance calculation of the route C bus passenger queue system at operating 09:45-10:45 hours

s	$\rho = \lambda/\mu s$	P_0	L_q	L	W_q	W
3	0.6909	0.0998	1.4123	3.4849	0.9353	2.3079
4	0.5181	0.1205	0.2727	2.3453	0.1806	1.5532
5	0.4145	0.1247	0.0634	2.136	0.042	1.4146
6	0.3454	0.1256	0.0147	2.0873	0.0097	1.3823

Table 16: The result of the performance calculation of the route C bus passenger queue system at operating 13:00-14:00 hours

s	$\rho = \lambda/\mu s$	P_0	L_q	L	W_q	W
2	0.7557	0.1392	0.3373	1.8486	0.43	2.3566
3	0.5038	0.2078	0.041	1.5523	0.0523	1.9789
4	0.3778	0.2184	0.0078	1.5191	0.0099	1.9365
5	0.3023	0.2203	0.0015	1.5128	0.0019	1.9285

4.4 Evaluation of Queue Performance

To find the optimal number of buses, weigh some things from the performance size of the queue system. For example, optimal queue system performance has a value of ρ about 85%. Not only that, the optimal queue system performance has small L_q, L, W_q, and W values. Based on the description, the optimal number of buses can be seen in the following table:

Table 17: Number of optimal buses

Bus routes	Number of buses at operating hours:					
	07:00 – 08:00		09:45 – 10:45		13:00 – 14:00	
	Reality	Optimal	Reality	Optimal	Reality	Optimal
A	5	5	3	2	3	2
B	5	5	3	3	3	2
C	6	6	3	3	3	2

5. Conclussion

Based on the calculation results, the model of the queue system formed is $(G/G/s):(FCFS/\infty/\infty)$. While the optimal number of buses so that the passenger queue system does not experience significant buildup can be seen in Table 17. Overall, the reality bus passenger queue system is optimal. Further studies can add cost factors such as bus procurement, maintenance costs, etc. The bulk service queue model can also be applied to this queue system.

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