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Comparison of Numerical Simulation of Epidemiological Model between Euler Method with 4th Order Runge Kutta Method

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Abstract

Coronavirus Disease 2019 has become global pandemic in the world. Since its appearance, many researchers in world try to understand the disease, including mathematics researchers. In mathematics, many approaches are developed to study the disease. One of them is to understand the spreading of the disease by constructing an epidemiology model. In this approach, a system of differential equations is formed to understand the spread of the disease from a population. This is achieved by using the SIR model to solve the system, two numerical methods are used, namely Euler Method and 4th order Runge-Kutta. In this paper, we study the performance and comparison of both methods in solving the model. The result in this paper that in the running process of solving it turns out that using the euler method is faster than using the 4th order Runge-Kutta method and the differences of solutions between the two methods are large.

Keywords: Euler method,4th order Runge-Kutta method, Epidemiology Model

1. Introduction

In the end of 2019, there is a novel worldwide outbreak of a new type of coronavirus (2019-nCoV)(Paolucci et al., 2020). It was first reported in Wuhan, China for the occurrence of the corona virus caused by Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2) or better known as Coronavirus Disease-19 (COVID-19). The disease has spread to other countries and has reached to 196 countries (Ahmed et al., 2020). The disease was first infected in humans when there was a person who was eating the bat soup at traditional market in Wuhan, China. Which was suspected to be the cause. Then, from one person the disease spread to the people through the air or in direct contact with the sufferer. The initial condition occurred coughing, flu, fever, and finally died, hence that all the city of Wuhan finally carried out a lockdown by the local government. Unfortunally, the lockdown did not stop the disease to spread throughout the country. In February there was the largest positive COVID-19 case in Italy with 235.270 cases, and on February 21st, 2020 worldwide there had been 7.151.000cases and 407.145 had died from coronavirus (Paolucci et al., 2020). Up to now there are 50 million of people infected with corona virus and 1.4 million of people had died from COVID-19, this data is taken based on per November 21st, 2020 (Ahmed et al., 2020; Grassin-Delyle et al., 2021).

The COVID-19has infected millions of people around the world and medical personnel as the vanguard is the main victims of the ferocity of the covid-19. It also disrupted the economies around the world. Now, people experience changes in their daily activities, including maintaining distance, washing hands, and wearing masks with the aim of preventing the spread of the COVID-19, because the SARS-CoV-2can be transmitted through the air and saliva. Various ways have been done by medical personnels and researchers to fight the COVID-19with vaccines but they were not obtained until the final stages of clinical trials.

To understand the disease, various diciplines are involved, including mathematics. In mathematics, a tool to know how the disease spreads, is called the epidemiological model. The SIR model is a simple dynamic model that describes the spread of disease between populations (Kolokolnikov & Iron, 2021). There are many researches using this approach (Xu et al., 2013; Lede and Mungkasi, 2019; Peng et al., 2020; Salim etal., 2020; Shao et al., 2020; Zhu and Zhu, 2020). The basic of the epidemiology model is SIR model that can be used to analyze the spreading of the disease between population. The model is consisted of some differential equations that can be solved by numerical methods.

In Mathematics, we are introduced with the term numerical solution of differential equation that can be used to approximate its analytical solution. This includes methods such as Euler and Runge-Kutta methods. Basically the Euler method is geometric intuition, taking it by small steps is the way to solve it for example$h > 0$, and approaching$y(t+h) \approx y(t) + hf\big(t, y(t)\big)$(Edalat et al., 2020). In 1768 the first time Leonhard Euler, Beethoven of mathematics, reported on the use of basic difference methods to obtain approximate solutions to differential equations or initial value problems (Biswas et al., 2013; Musa et al., 2010). Basically, Euler method uses its slope at a point to find the numerical solution near that point. While the Runge-Kutta method is an extension modification of the Euler method which has a higher level of accuracy. The Runge-Kutta method was developed by German mathematical scientists Carl Runge and Martin Wilhelm Kutta. The Runge-Kutta method has similarities with the Euler method, which is close to the number at each point. However, the Runge-Kutta has a larger number of slope weights at each time, so it is more accurate than the Euler method (Molthrop, 2018).In numerical analysis, the Runge-Kutta formula is one of the oldest and best understood designs (Musa et al., 2010; Vaidyanathan et al., 2017; Vaidyanathan et al., 2018).

In this study, we aim to compare the results of both Euler and Runge-Kutta methods appliedon the problem of spread of the Covid-19, in terms of the errors and running time of computer programming.

2. Materials and Methods

2.1. Materials

In this paper, epidemiological dynamic model SIR is used to compare Euler method and 4th order Runge-Kutta. The model is solved using Euler method and 4th order Runge-Kutta by Python. The results can be described in the form of a curve and in the form of a table in the interval. The results can compare the diffrence of each other running time and the solutions.

2.2. Methods

2.2.1. SIR Model

Research There is an incidence of disease in a population that causes transmission from one patient to another so that it will result in interrelation with one another. In this model, we use Roda et al.(2020) model to describle the spread of COVID-19. The model is divided into three compartments, namely:

(i) Susceptible population $\big(S(t)\big)$
(ii) Infected population $\big(I(t)\big)$
(iii) Confirmed population $\big(R(t)\big)$

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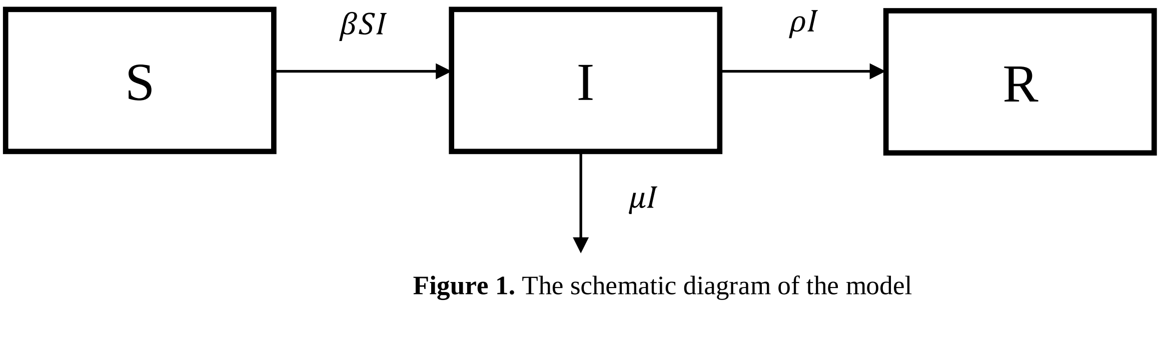

Figure 1. The schematic diagram of the model

The parameters of the model are described in Table 1.

Table 1. Parameter of the model.

Parameter	Description
β	Local transmission coefficient
ρ	Diagnosis rate
μ	The recovering rate of infected population

The dynamical equations are:

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - (\rho + \mu)I \quad (1)$$
$$\frac{dR}{dt} = \rho I$$

2.2.2. Euler Method

The Euler method is the basic method of solving a differential equation in the first order. To approach the solution of differential equations in the Euler method, namely by looking at the slope in a point (see Figure 2) (Molthrop, 2018). Incidentally, in 1714 Taylor's theorem was published, most likely with the aim of approaching $y(t+h)$ in writing known as $y(t)$ and its derivatives. Another thing in the concept of the Eulerian method can be seen from the merging of several small line segments that approach the actual curve of $y\ (t)$ versus t, the use of this concept is known as the concept of local linearity (Biswas et al., 2013). The common form of the euler method formula can be seen as follows:

$$y_{n+1} = y_n + \Delta t f(t, y(t)), \text{for } n = 0,1,2,...$$

Euler method used to solve system of nonlinear equations

$$\frac{dS}{dt} = f(t, S, I, R)$$
$$\frac{dI}{dt} = g(t, S, I, R) \quad (2)$$
$$\frac{dR}{dt} = j(t, S, I, R)$$

with starting value $S(0) = S_0, I(0) = I_0$ and $R(0) = R_0$with *stepsize* $h = t_{n+1} - t_n$are

$$S_{n+1} = S_n + hf(t_n, S_n, I_n, R_n)$$
$$I_{n+1} = I_n + hg(t_n, S_n, I_n, R_n) \quad (3)$$
$$R_{n+1} = R_n + hj(t_n, S_n, I_n, R_n)$$

By replacing the nonlinear equation with the SIR model in (1) known, the equation can be written as:

$$\frac{dS}{dt} = f(t, S, I, R) = -\beta SI$$
$$\frac{dI}{dt} = g(t, S, I, R) = \beta SI - (\rho + \mu)I \quad (4)$$
$$\frac{dR}{dt} = j(t, S, I, R) = \rho I$$

Using the euler method, the solution of the equation is

$$\begin{aligned} S_{n+1} &= S_n + h(-\beta S_n I_n) \\ I_{n+1} &= I_n + h(\beta S_n I_n - (\rho + \mu) I_n) \\ R_{n+1} &= R_n + h(\rho I_n) \end{aligned} \tag{5}$$

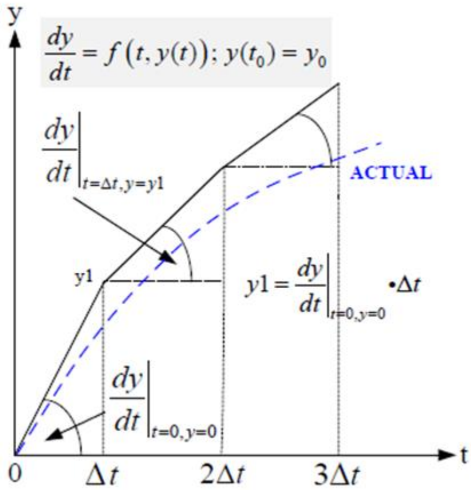

Figure 2. Ilustration of Euler Algorithm

The definition of the derivative is the basis of the Euler method. The slope of the y versus t curve can be represented by the form dy/dt. In solving the slope problem of $f(t, y(t))$ with the ordinate $y1$ can be do done by multiplying the slope by the equidistant step size (Biswas et al., 2013). As for the easier calculations per time step, it can be used with the explicit euler method, so that if implemented it can be relatively easy while the implicit method in other respects is relatively more difficult, which so it can be computationally intensive(Mid and Dua, 2019).

2.2.3. 4th Order Runge-Kutta Method

The most popular numerical approach in terms of accuracy, stability, and can be easily programmed is the Runge-Kutta method (Salim et al., 2020). The general forth order differential equations (ODEs) of the form $y^{(4)} = f(x, y, y', y'', y''')$, $0 \le x \le L$, can be solved by reducing it to its equivalent first order system as mentioned (Ahamad and Charan, 2019). Here is a common form of the4th order Runge-Kutta method

$$\begin{aligned} k_1 &= hf(t_i, u_i) \\ k_2 &= hf\left(t_i + \frac{h}{2}, u_i + \frac{1}{2}k_1\right) \\ k_3 &= hf\left(t_i + \frac{h}{2}, u_i + \frac{1}{2}k_2\right) \\ k_4 &= hf(t_{i+1}, u_i, k_3) \\ u_{i+1} &= u_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \qquad \text{for } i = 0, 1, \dots, n-1 \end{aligned}$$

They can be used to solve the first-order initial-value problem given below:

$$\frac{du}{dt} = f(t, u), \quad a \le t \le b, \;\; u(a) = \alpha.$$

The Runge Kutta method of order 4 turns into Simpson's rule for numerical integration on $[t_i, t_{i+1}]$, when there is no dependency f on u. The Runge-Kutta method can be considered to act similarly to the Euler method, where it uses the weighted sum of the slope for each time and value of the function to obtain a more accurate estimate (Molthrop, 2018). Here is the 4th order Runge-Kutta method written in different notations

$$\begin{aligned} k_1 &= F(t, y_n) \\ k_2 &= F\left(t + \frac{h}{2}, y_n + \frac{k_1}{2}\right) \\ k_3 &= F\left(t + \frac{h}{2}, y_n + \frac{k_2}{2}\right) \\ k_4 &= F(t + h, y_n) \\ y_{n+1} &= y_n + \frac{\mathrm{h}}{6}(k_1 + 2k_2 + 2k_3 + k_4), \qquad for\ \ n = 0,1,2,\dots \end{aligned} \tag{6}$$

To solve the SIR model (1), the equation can be written as

$$\frac{dy}{dt} = F(t, y) \tag{7}$$

with

$$y = \begin{pmatrix} S(t) \\ I(t) \\ R(t) \end{pmatrix}, F(t, y) = \begin{pmatrix} f(t, S, I, R) \\ g(t, S, I, R) \\ j(t, S, I, R) \end{pmatrix} \tag{8}$$

3. Results and Discussion

In this paper, we simulate our model with data case in Wuhan, China fromRoda et al. (2020)research Molthrop (2018). It is assumed that the values of the parameters for the SIR model (1) are $\beta = 9.906e - 8$, $\rho = 0.24$ and $= 0.1$, with the starting value $S_0 = 6e + 6$, $I_0 = 245$, $R_0 = 0$. The stepsize we choose to simulate the model is $h = 0.1$. The solution of the model using the Euler method with a value of $h = 0.1$, is as follow:

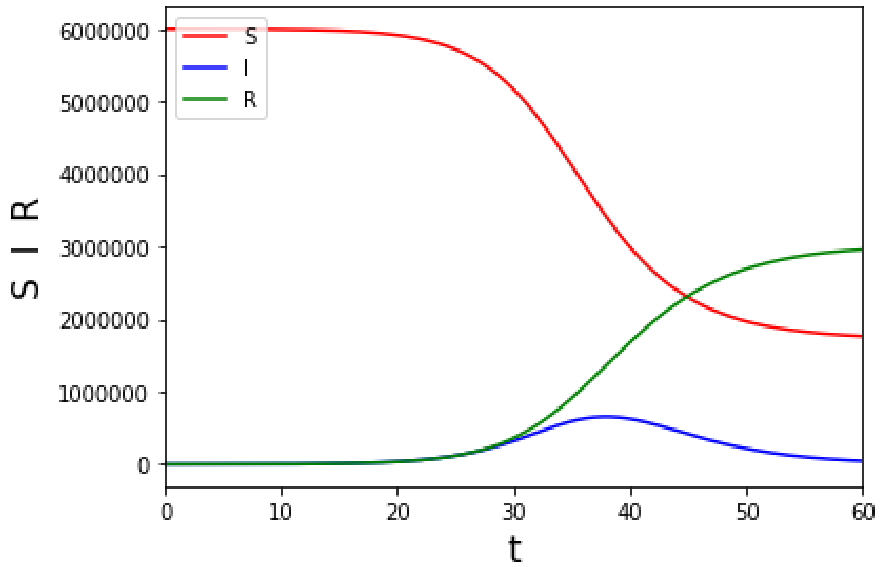

Figure 3: SIR model solution with Euler method

The running time required to complete the SIR model is 0.09375 second using Python. Whereas the solution of the SIR model using 4th order Runge-Kutta method is as follows:

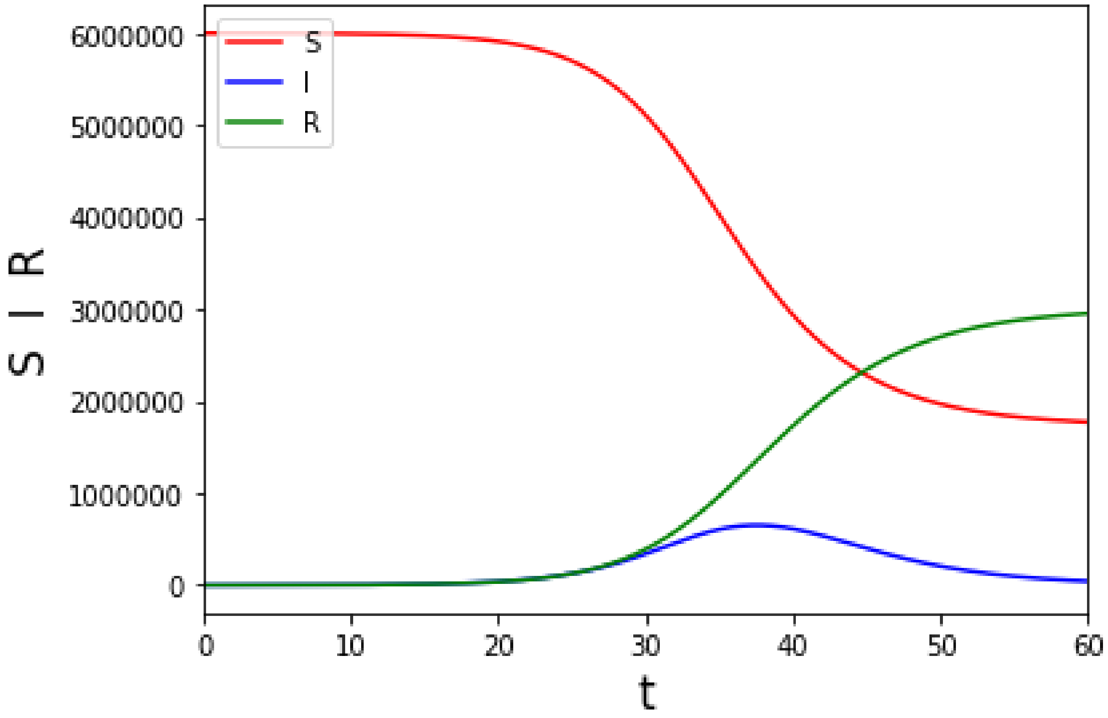

Figure 4: SIR model solution with 4^{th} order Runge-Kutta method

The running time required to complete the SIR model is 0.203125 second using Python.The absolute valuedifference of the solution with the Euler method and the 4^{th} order Runge-Kuttamethod is as follows:

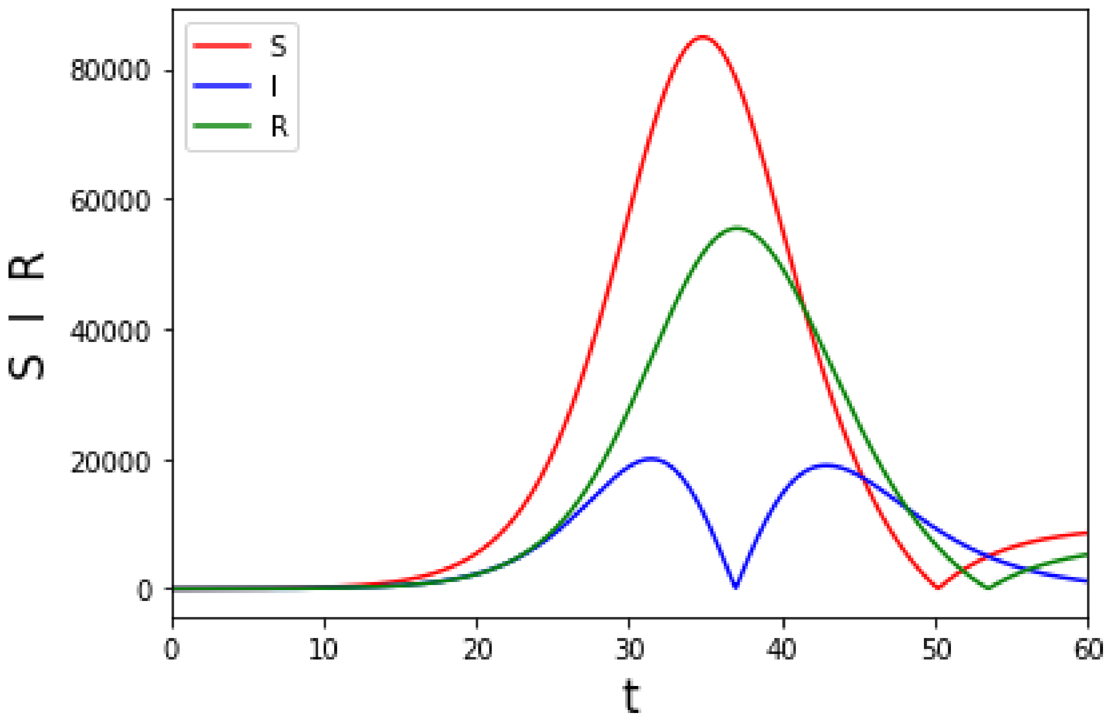

Figure 5: The absolute value difference of the SIR model solution between two method

From Figure 5, we can find out that the absolute value differences are small at the beginning. But when the value of increase, the absolute value differences are increasing too until their peak and it decrease over time. The absolute value differences of both solutions are large, expecially in interval [20,50]. To find out more absolute value differences of the solution in interval [20,50], in this paper we also show the solution of the model using Euler method and 4^{th} order Runge-Kutta method time interval [20,50] in Table 2**.**

Table 2. The Solutions and Differences from Euler and Runge Kutta 4th order methods

Time	S(t) Euler	S(t) RK	abs dif S(t)	I(t) Euler	I(t) RK	abs dif I(t)	R(t) Euler	R(t) RK	abs difR(t)
20	5915474	5910085	5389.824	36078.76	38335.54	2256.777	34370.68	36582.24	2211.563
21	5891881	5884712	7169.172	45958.22	48941.41	2983.187	44051.09	47005.9	2954.813
22	5861996	5852523	9472.416	58394.22	62304.38	3910.164	56368.17	60294.47	3926.296
23	5824294	5811874	12420.6	73956.95	79031.52	5074.571	71995.52	77180.95	5185.43
24	5776975	5760832	16142.94	93288.59	99796.92	6508.333	91751.66	98552.56	6800.898
25	5717963	5697199	20764.02	117077.9	125307.1	8229.156	116615	125463.1	8848.136
26	5644951	5618569	26382.04	146009.8	156236.6	10226.72	147730.1	159133.8	11403.76
27	5555499	5522463	33036.8	180681.1	193125.6	12444.55	186398.5	200934.2	14535.71
28	5447207	5406538	40668.19	221475.1	236234.9	14759.76	234044.7	252333	18288.3
29	5317971	5268900	49071.36	268397.6	285364.2	16966.67	292148.1	314810.3	22662.13
30	5166337	5108476	57861.35	320887	339660.8	18773.76	362132.1	389723.4	27591.24
31	4991894	4925428	66466.2	377639.7	397465.8	19826.11	445208	478130.5	32922.42
32	4795645	4721476	74168.61	436506.5	456268.3	19761.75	542183.9	580588.7	38404.84
33	4580256	4500051	80205.04	494533.8	512832.3	18298.47	653262.5	696961.2	43698.75
34	4350064	4266155	83908.54	548199.4	563528.5	15329.1	777869.5	826278.6	48409.02
35	4110775	4025920	84855.01	593843.1	604831.4	10988.27	914560.5	966701.7	52141.23
36	3868885	3785924	82960.68	628212.7	633868.4	5655.717	1061045	1115613	54568.21
37	3630938	3552446	78492.71	648985.7	648870.6	115.1228	1214344	1269832	55487.88
38	3402801	3330809	71991.54	655117.1	649397.2	5719.874	1371055	1425910	54855.12
39	3189123	3124983	64139.29	646922.8	636291.6	10631.26	1527670	1580449	52779.21
40	2993076	2937452	55623.82	625901.6	611412.5	14489.09	1680895	1730386	49491.46
41	2816360	2769323	47037.07	594377	577245	17131.94	1827888	1873184	45295.77
42	2659403	2620580	38823.13	555079.2	536502.5	18576.74	1966420	2006938	40517.55
43	2521665	2490395	31270.23	510766.5	491798.7	18967.75	2094927	2130389	35462.11
44	2401950	2377418	24531.16	463946.7	445428.5	18518.15	2212482	2242869	30387.75
45	2298680	2280024	18655.71	416714.4	399254.2	17460.18	2318718	2344212	25493.57
46	2210116	2196492	13623.94	370689.6	354678.9	16010.66	2413722	2434640	20918.54
47	2134502	2125127	9374.161	327028.5	312676.5	14352.04	2497916	2514664	16747.91
48	2070160	2064337	5823.712	286478.9	273853.2	12625.69	2571957	2584980	13023.11
49	2015550	2012667	2883.05	249453.8	238520.8	10932.98	2636641	2646394	9752.495
50	1969285	1968821	464.405	216109.2	206768.5	9340.722	2692836	2699757	6921.266

From Table 2, we can find that the largest absolute value difference of the solution of $S(t)$ in interval [0,60] is 84855.01 when $t = 35$, the largest absolute value difference of the solution of $I(t)$ in interval [0,60] is 19826.11 when $t = 31$, and the largest absolute value difference of the solution of $R(t)$ in interval [0,60] is 55487.88 when $t = 37$. From the data, we can conclude that the absolute value differences between two method are large.

4. Conclussion

The Euler method and 4th order Runge-Kutta method can be used to solve SIR Model. The solutions of the methods are approximations to the exact solutions. The differences between the Euler method and Runge-Kutta 4th are significant. The running time of Euler method is shorter than that of 4th order Runge-Kutta method. The differences of solutions between two methods are large at interval [20,50].

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