Optimization of the Mean-Variance Model Investment Portfolio in Five Mining Stocks Traded on the IDX

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Abstract

The Mining and Energy sector is a major foreign exchange earner, provides the largest energy resource, and as an absorber of labor. In addition, most of the energy resources used in the Indonesian economy come from mining, namely oil and coal. Investment for mining and energy exploration in Indonesia needs to be a priority and continue to be encouraged to maintain the level of reserves as raw materials for future industrial development, including downstream. This study aims to measure the performance of investment portfolios in several stocks in the Mining and Energy sectors. The portfolio optimization method is carried out using the Mean-Variance model (Markowitz model). Based on the results of the analysis, it is obtained that the combination and proportion of capital allocation on several stocks in the formation of an investment portfolio that has better performance, where the optimum portfolio composition obtained a portfolio return of 0.000866205 with a portfolio variance of 0.000261104. In addition, the results of the analysis can be concluded that the return ratio can affect the model.

Keywords: mining and energy stocks, Markowitz model, optimum portfolio

1. Introduction

The Mining and Energy sector is the main producer of foreign exchange, provides the largest energy resource, and is an absorber of labor. In addition, most of the energy resources used in the Indonesian economy come from mining, namely oil and coal. Thanks to the availability of mining products, Indonesia does not need to import energy resources most of what is produced can be exported. One of the uses of energy resources is to generate electricity which is indispensable for the development of other sectors. The potential for developing the Mining and Energy Sector is quite large, but the challenges and problems faced by this sector are quite significant. Since most of the mining products are produced solely for foreign markets, the development of this sector is strongly influenced by price fluctuations in the market. For this reason, it is necessary to invest in mining and energy exploration in Indonesia so that it becomes a priority and continues to be encouraged to maintain the level of reserves as raw materials for future industrial development.
Even today, investment is very important to improve economic welfare, it is one of the most important forms of economic activity that individuals, businesses, and governments do. Investment is a commitment made at this time on several funds or resources to achieve a profit (Panjer, 1998; Tsay, 2005; Tandelilin, 2010). Then according to Martono (2002) and Jogiyanto (2003), investment is the investment of funds made by a company into an asset (assets) in the hope of obtaining future income. Meanwhile, Husnan (2006), stated that project investment is a plan to invest resources, both giant projects or small projects to obtain benefits in the future, generally these benefits are in the form of value for money. Meanwhile, capital can be in the form of non-money, for example, land, machinery, buildings, and others. So, the investment itself is an investment made by investors in the form of funds or materials that have a long period in the hope of getting profits in the future.

In making investments, investors need to pay attention to what will be invested, how much amount they want to invest, and the level of risk that investors are ready to bear to achieve their investment goals. A portfolio is simply a collection of financial assets that are investment tools such as bonds, foreign exchange, stocks, gold, real estate certificates, bank deposits, and other instruments that are simultaneously owned by individuals or groups.

All portfolio investments contain an element of uncertainty risk, for that, it is necessary to conduct a scientific analysis first so that they can choose investments that are truly safe and provide optimal returns (Logubayom and Victor, 2020). According to Rahman and Adnan (2020), the risk is the possibility of storing the actual level of profit (actual return) from the level of expected profit (expected return). Investment risk is best understood when expressed in statistical terms that consider the entire range of possible returns on investment. Markowitz stated that the expected return (mean), variance, and standard deviation (risk) of a portfolio are the overall criteria for portfolio selection and construction. This parameter can be used as a proverb/advice on how investors need to act. It is interesting to note that the entire model is based on the economic facts of “Expected Utility”. The concept of utility here is based on the fact that different investors have different investment goals and abilities, being satisfied in different ways (Qur’anitasari et al., 2019).

Markowitz (1952) developed a modernization for portfolios, Markowitz used the set approach of portfolio efficiency represented as a combination of two separate stocks. After Markowitz's discovery, Sharpe (1964) and Litner (1965) found the Capital Asset Pricing Model (CAPM) to evaluate Markowitz's portfolio performance. Maf’ula et al. (2018) conducted a comparative study of the optimal portfolio of the Markowitz portfolio development model, namely Mean-Variance (MV), Downside Deviation (DD), and Mean Absolute Deviation (MAD) investments with a study on the BISNIS-27 index on the Indonesia Stock Exchange (IDX). The results of this study explain that the MAD model is an optimal portfolio model that can provide high returns and optimal performance, so it is appropriate for investors with risk seeker preferences. The DD model is an optimal portfolio model that can provide the least risk, so this model is appropriate for investors with risk-averse preferences.

After knowing the form of a portfolio based on the Markowitz model, investors can assess the performance of the portfolio. The purpose of this activity is to increase the likelihood of achieving investors' investment goals. Performance appraisal is a measure for investors in making decisions about an investment (Aprilia et al., 2018).

From the explanation above, the purpose of this research is to create a portfolio optimization model that can help investors in the investment process and measure the performance of investment portfolios. Because that a portfolio that provides a greater rate of return, does not always have a better performance than other portfolios. This is due to the importance of considering risk factors so that there is a need for standard portfolio performance measurements. From this objective, investors are also expected to obtain the maximum possible return with minimum risk. The method that will be used is the Markowitz model. The implications of this research can assist investors to consider the selected asset when investing.
2. Materials and Methods

2.1. Materials

The material for optimizing the investment portfolio of the Mean-Variance model or Markowitz model is from the analyzed data consisting of 10 selected mining and energy sector stocks, which include stock prices: ADRO, ANTM, BSSR, DKFT, ELSA, PGAS, PSAB, RUIS, SMRU, and TINS. The stock price data is the daily transaction value, for the period April 5, 2016 to April 5, 2019, traded on the Indonesia Stock Exchange (IDX), which is accessed through the website: www.yahoofinance.com. Data analysis was carried out with the help of MS Excel and Matlab software.

2.2. Methods

Optimizing the investment portfolio of the Mean-Variance model is done by selecting the 5 best stocks as seen from the expected value/average return divided by the variance. Furthermore, the Mean-Variance model is used to analyze the investment portfolio in 5 selected mining and energy sector stocks. In this investment portfolio analysis, efficient graphs and optimum portfolios are shown.

3. Results and Discussion

3.1. Descriptive statistics

Referring to the discussion of materials in section 2.1, that the data analyzed is data consisting of 10 selected mining and energy sector stocks, which include stock prices: ADRO, ANTM, BSSR, DKFT, ELSA, PGAS, PSAB, RUIS, SMRU, and TINS. Furthermore, the price of these shares is determined by the value of the return on each share based on the principle of calculating stock returns. The calculation of stock returns is carried out with the help of MS Excel software, and these stock returns are determined by descriptive statistical values which include: expectation/average, variance, and ratio. The results of the descriptive statistical calculations are given in Table 1.

<table>
<thead>
<tr>
<th>Stock Name</th>
<th>Expectation/Average (µ)</th>
<th>Variance(σ²)</th>
<th>Ratio ($\frac{µ}{σ^2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADRO</td>
<td>0.000829947</td>
<td>0.000731517</td>
<td>1.34554716</td>
</tr>
<tr>
<td>ANTM</td>
<td>0.000779667</td>
<td>0.000704600</td>
<td>1.106538645</td>
</tr>
<tr>
<td>BSSR</td>
<td>0.000854911</td>
<td>0.001116855</td>
<td>0.765463305</td>
</tr>
<tr>
<td>DKFT</td>
<td>-3.21414E-18</td>
<td>0.000444779</td>
<td>-7.22637E-15</td>
</tr>
<tr>
<td>ELSA</td>
<td>0.000118866</td>
<td>0.000844052</td>
<td>0.140827949</td>
</tr>
<tr>
<td>PGAS</td>
<td>-0.000242925</td>
<td>0.000872325</td>
<td>-0.278479417</td>
</tr>
<tr>
<td>PSAB</td>
<td>-0.000175468</td>
<td>0.000999263</td>
<td>-0.175597712</td>
</tr>
<tr>
<td>RUIS</td>
<td>0.000227414</td>
<td>0.000523631</td>
<td>0.434302064</td>
</tr>
<tr>
<td>SMRU</td>
<td>0.001011078</td>
<td>0.001435154</td>
<td>0.704508552</td>
</tr>
<tr>
<td>TINS</td>
<td>0.000842023</td>
<td>0.000804495</td>
<td>1.046646854</td>
</tr>
</tbody>
</table>

Looking at the descriptive statistical values given in Table 1, it can be seen that of the 10 mining and energy sector stocks analyzed, they have different expectations/means and variances. The smallest expected/average return value is owned by PGAS shares, which is -0.000242925 with a variance of 0.000844052, and the largest expected/average return is owned by SMRU shares, which is 0.001011078 with a variance of 0.001435154. It appears that stocks that have a small expectation/average return are followed by a small variance (risk), on the other hand, stocks that have a large expectation/average return are followed by a large variance (risk). This shows that in investing in financial assets, an asset that promises a greater return will be followed by a greater risk that investors must face.
3.2. Portfolio Optimization Process

Furthermore, from the expected/average return values from Table 1, it is used to form the average return $\mu^T$ vector as follows

$$
\mu^T = (0.000829947 \ 0.000779667 \ 0.000842023 \ 0.000854911 \ 0.001011078),
$$

and because 5 stocks are analyzed, the unit vector $I^T$ of its elements consists of the number one as much as 5 as given here $I^T = [1 \ 1 \ 1 \ 1 \ 1]$ (Kalfin et al., 2019). The stock return variance values, together with the covariance values between stock returns, are used to form the variance-covariance matrix as follows

$$
\Sigma = \begin{bmatrix}
0.0007315 & 0.0001989 & 0.0002203 & 0.0000812 & -0.0000655 \\
0.0001989 & 0.00007046 & 0.0003730 & 0.0000327 & 0.0000143 \\
0.0002203 & 0.0003730 & 0.0008045 & -0.0000252 & -0.0000379 \\
0.0000812 & 0.0000327 & -0.0000252 & 0.0011169 & 0.0000838 \\
-0.0000655 & 0.0000143 & -0.0000379 & 0.0000838 & 0.0014352
\end{bmatrix}
$$

and the inverse of the covariance matrix $\Sigma^{-1}$ as follows

$$
\Sigma^{-1} = \begin{bmatrix}
1.55113 & -0.27690 & -0.29662 & -0.11680 & 0.07254 \\
-0.27690 & 1.93863 & -0.82701 & -0.05148 & -0.05079 \\
-0.29662 & -0.82701 & 1.71189 & 0.08177 & 0.03514 \\
-0.11680 & -0.05148 & 0.08177 & 0.91137 & -0.05587 \\
-0.07254 & -0.05079 & 0.03514 & -0.05587 & 0.70477
\end{bmatrix}
$$

The vectors $\mu^T$ and $I^T$, as well as the matrix $\Sigma^{-1}$, are then jointly used to determine the optimum weight of the investment portfolio. The risk tolerance $\tau$ with the condition $\tau \geq 0$ in the optimization of the investment portfolio is simulated by taking several values that meet the conditions of $e^T w = 1$. Taking the risk tolerance value is stopped, if for a risk tolerance value it produces a weight $w_i (i = 1, ..., 5)$ which is not a positive real number and meets $e^T w = 1$. To simplify the calculation, Matlab software is used, the results of the investment portfolio optimization process are summarized and presented in Table 2.

**Table 2: Summary of Investment Portfolio Optimization Process Results**

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>ADRO</th>
<th>ANTM</th>
<th>TINS</th>
<th>BSSR</th>
<th>SMRU</th>
<th>$w^T e$</th>
<th>$\mu_p$</th>
<th>$\sigma_p^2$</th>
<th>$\frac{\mu_p}{\sigma_p^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.2426</td>
<td>0.1904</td>
<td>0.1833</td>
<td>0.1999</td>
<td>0.1835</td>
<td>1</td>
<td>0.0008</td>
<td>0.0002</td>
<td>3.3104</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2418</td>
<td>0.1764</td>
<td>0.1882</td>
<td>0.1992</td>
<td>0.1942</td>
<td>1</td>
<td>0.0008</td>
<td>0.0002</td>
<td>3.3174</td>
</tr>
<tr>
<td><strong>0.2</strong></td>
<td><strong>0.2410</strong></td>
<td><strong>0.1623</strong></td>
<td><strong>0.1931</strong></td>
<td><strong>0.1984</strong></td>
<td><strong>0.2050</strong></td>
<td><strong>1</strong></td>
<td><strong>0.0008</strong></td>
<td><strong>0.0002</strong></td>
<td><strong>3.3174</strong></td>
</tr>
<tr>
<td>0.3</td>
<td>0.2402</td>
<td>0.1482</td>
<td>0.1980</td>
<td>0.1976</td>
<td>0.2157</td>
<td>1</td>
<td>0.0008</td>
<td>0.0002</td>
<td>3.3107</td>
</tr>
<tr>
<td>0.4</td>
<td>0.2393</td>
<td>0.1342</td>
<td>0.2029</td>
<td>0.1969</td>
<td>0.2265</td>
<td>1</td>
<td>0.0008</td>
<td>0.0002</td>
<td>3.2972</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2385</td>
<td>0.1201</td>
<td>0.2078</td>
<td>0.1961</td>
<td>0.2372</td>
<td>1</td>
<td>0.0008</td>
<td>0.0002</td>
<td>3.2774</td>
</tr>
<tr>
<td>0.6</td>
<td>0.2377</td>
<td>0.1061</td>
<td>0.2126</td>
<td>0.1954</td>
<td>0.2479</td>
<td>1</td>
<td>0.0008</td>
<td>0.0002</td>
<td>3.2514</td>
</tr>
<tr>
<td>0.7</td>
<td>0.2369</td>
<td>0.0920</td>
<td>0.2175</td>
<td>0.1946</td>
<td>0.2587</td>
<td>1</td>
<td>0.0008</td>
<td>0.0002</td>
<td>3.2196</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2360</td>
<td>0.0780</td>
<td>0.2224</td>
<td>0.1939</td>
<td>0.2694</td>
<td>1</td>
<td>0.0008</td>
<td>0.0002</td>
<td>3.1824</td>
</tr>
<tr>
<td>0.9</td>
<td>0.2352</td>
<td>0.0639</td>
<td>0.2273</td>
<td>0.1931</td>
<td>0.2802</td>
<td>1</td>
<td>0.0008</td>
<td>0.0002</td>
<td>3.1402</td>
</tr>
<tr>
<td>1.0</td>
<td>0.2344</td>
<td>0.0499</td>
<td>0.2322</td>
<td>0.1924</td>
<td>0.2909</td>
<td>1</td>
<td>0.0008</td>
<td>0.0002</td>
<td>3.0936</td>
</tr>
<tr>
<td>1.1</td>
<td>0.2336</td>
<td>0.0358</td>
<td>0.2371</td>
<td>0.1916</td>
<td>0.3017</td>
<td>1</td>
<td>0.0008</td>
<td>0.0002</td>
<td>3.0430</td>
</tr>
<tr>
<td>1.2</td>
<td>0.2327</td>
<td>0.0217</td>
<td>0.2420</td>
<td>0.1909</td>
<td>0.3124</td>
<td>1</td>
<td>0.0008</td>
<td>0.0002</td>
<td>2.9889</td>
</tr>
<tr>
<td>1.3</td>
<td>0.2319</td>
<td>0.0077</td>
<td>0.2469</td>
<td>0.1901</td>
<td>0.3232</td>
<td>1</td>
<td>0.0008</td>
<td>0.0003</td>
<td>2.9319</td>
</tr>
<tr>
<td><strong>1.4</strong></td>
<td><strong>0.2311</strong></td>
<td><strong>-0.0063</strong></td>
<td><strong>0.2518</strong></td>
<td><strong>0.1894</strong></td>
<td><strong>0.3339</strong></td>
<td><strong>1</strong></td>
<td><strong>0.0008</strong></td>
<td><strong>0.0003</strong></td>
<td><strong>2.8723</strong></td>
</tr>
</tbody>
</table>
Considering Table 2, taking the risk tolerance value is only for the value $0 \leq \tau \leq 1.3$, with an increase of 0.1. This is because the risk tolerance value $\tau > 1.3$ produces a negative weight. If it is assumed that short sales are not allowed, then the investment portfolio weights with negative values do not need to be analysed again (Qin, 2015). In the optimization process, the composition of the investment portfolio weights on 5 stocks with different values. This results in different values of expected/average portfolio return ($\mu_p$) and portfolio variance ($\sigma_p^2$), as shown in Table 2.

### 3.3. Discussions

The discussion in this section is more related to the preferences described by the level of risk of each investor, which in this study is assumed that the investor concerned invests in 5 shares of the mining and energy sectors. In this case, the risk tolerance level $\tau$ is described which lies in the interval $0 \leq \tau \leq 1.3$. Based on the level of risk tolerance in the interval $0 \leq \tau \leq 1.3$ with an increase of 0.1, and using the values given in Table 2, a surface graph of the efficiency of the investment portfolio can be made, as given in Figure 1.

![Figure 1: Investment Portfolio Efficiency Surface Graph](image)

For each different level of risk tolerance, it produces a different magnitude of expected/average return of the $\mu_p$ portfolio and the $\sigma_p^2$ portfolio risk. For rational investors, with their level of risk tolerance, of course, they will invest at points along the surface of an efficient portfolio. For investors who invest beyond the points along the surface line of an efficient portfolio, it can be viewed as an irrational investor (Wang et al., 2016; Shakouri and Lee, 2016). Along the efficient portfolio surface, a portfolio that has a minimum Value-at-Risk (VaR) risk occurs when the risk tolerance level $\tau = 1.3$ which results in the expected/average return of the portfolio $\mu_p = 0.00089583$ and portfolio risk $\sigma_p^2 = 0.000305542$ with a ratio value $\frac{\mu_p}{\sigma_p^2} = 2.93193604953044$ is the smallest, so it is often referred to as the minimum portfolio. This minimum portfolio is generated if the investment is made with $w^{Min} = (0.23197287 \ 0.00773421 \ 0.24691850 \ 0.190165022 \ 0.32320940)^T$ weight vector composition. For the risk tolerance level in the interval $0 \leq \tau \leq 1.3$ with an increase of 0.1, the maximum portfolio occurs when the risk tolerance level $\tau = 0.2$ which produces the expected/average return of the portfolio $\mu_p = 0.000866205$ and the portfolio risk $\sigma_p^2 = 0.000261104$ with the ratio value.
\( \frac{\mu_p}{\sigma_p} = 3.31747006464924 \) is the largest, so it is often referred to as the maximum portfolio. This maximum portfolio is generated if the investment is made with \( \mathbf{w}^{M} = (0.2410489, 0.1623476, 0.1931385, 0.1984513, 0.2050136)^T \) weight vector composition.

Next, if we look at the relationship between the \( \sigma_p^2 \) risk variance and the expected/average return of the \( \mu_p \) portfolio, we can observe the graph given in Figure 2.

Looking at Figure 2, it can be seen that the ratio between expectations/average and the largest portfolio return variance is 3.31747006464924 or is obtained when the risk tolerance is \( \tau = 0.2 \). The ratio between expectation/average and variance of portfolio return increased at risk tolerance interval \( 0 \leq \tau < 0.2 \) and decreased at risk tolerance interval \( 0.2 < \tau \leq 1.3 \). The complete numerical results are presented in Table 2. From the graph of the relationship between the risk variance \( \sigma_p^2 \) and the expected/average return of the \( \mu_p \) portfolio, which the graph has the expected value/average return of the \( \mu_p \) portfolio which tends to decrease along with the increase in the value of the \( \sigma_p^2 \) risk variance. In investing in financial assets in general, assets that have a high level of risk tolerance will also provide high return expectations. On the other hand, assets that promise a small expected return will generally be accompanied by a small risk.

4. Conclusion

This paper discusses investment optimization in five mining and energy sector stocks using the Mean-Variance model. From the results of data analysis of 5 stocks, an efficient portfolio surface graph has been formed with a minimum expected/average portfolio return value of 0.000860819 and a minimum portfolio risk \( \sigma_p^2 \) of 0.000260027, occurring for a risk tolerance level of 0.00. In this analysis, the portfolio surface graph has a maximum expected/average portfolio return value of 0.00089583 and a maximum portfolio risk of 0.000305542, which occurs when the risk tolerance level is 1.3. In addition, the results of the discussion show that the greater the level of risk tolerance, the smaller the risk value of the VaR portfolio, followed by the smaller the expected value/average return of the portfolio. As for the optimum portfolio in Table 2, it can be seen that the optimum portfolio is composed of 5 stocks with \( \mathbf{w}^* = (0.2410489, 0.1623476, 0.1931385, 0.1984513, 0.2050136) \) weight vector composition,
sequentially for ADRO, ANTM, TINS, BSSR, and SMRU stocks. This optimal portfolio composition produces an average return of 0.000866205 and a variance of 0.000261104.

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**References**


